This is the homework for the week of August 31 through September 3, covering Chapter 1, Sections 1 to 3 (but mostly Section 1). It is due in class on Wednesday, September 9. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit. You are encouraged to come in for help on doing the homework and on understanding the material, if you need it.

1. Draw truth tables to prove the following logical equivalences:

a) $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ (One of DeMorgan's Laws)

b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

2. In each of the following pairs of propositions, one of the propositions is a tautology. Which one? (Justify your answer!)

a) $(p \vee q) \rightarrow (p \wedge q)$, $(p \wedge q) \rightarrow (p \vee q)$

b) $p \rightarrow (p \wedge q)$, $p \rightarrow (p \vee q)$

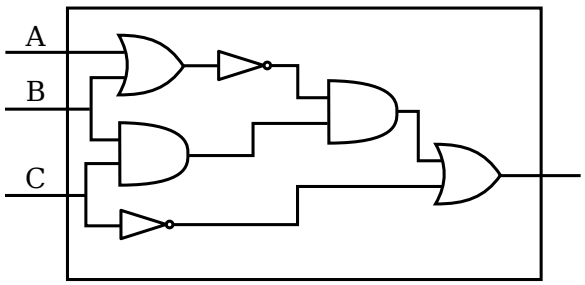
c) $p \rightarrow (p \rightarrow q)$, $p \rightarrow (q \rightarrow p)$

3. Is the operator "$\rightarrow$" associative? That is to say, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Is the operator "$\leftrightarrow$" associative? (Justify your answers!)

4. Let's say that a logical operator $\uparrow$ is defined as follows: For any propositions A and B, the expression $A \uparrow B$ has the same logical value as the expression $\neg(A \wedge B)$. (In computer science, this operator is usually called `NAND`, which is short for "not and.") For each of the three logical expressions $\neg p$, $p \wedge q$, and $p \vee q$, show how to write an equivalent expression that uses no logical operator other than $\uparrow$. (You might need some hints: The expression that is equivalent to $\neg p$ contains p **twice**. Once you know how to write $\neg p$ in terms of $\uparrow$, you can use that to help you with $p \wedge q$ and $p \vee q$.) (Don't just give an answer; explain why your expressions are equivalent to the original expressions.)

5. Design and draw a logic circuit that has three inputs and one output with the following property: The output is on if and only if the values of the three inputs are all the same. (Explain your reasoning!)

6. Find the compound proposition computed by the following logic circuit:



7. Convert each of the following English statements into propositional logic. You should introduce symbols such as p and q to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for. The first statement is *ambiguous*; you should give two possible translations and explain the difference.

a) I like pizza with mushrooms and sausage or black olives.

b) He is neither smart nor lucky.

c) If you are smart or lucky, you will be rich and famous.

8. Express the negation of each of the following sentences in natural, unambiguous English.

a) CPSC 229 is not meant to be fun.

b) The answer is greater than five and less or equal to than ten.

c) If Fred lives to be 100, he will still be a grouch.

9. **a)** Find **(1)** the *converse*, **(2)** the *contrapositive*, and **(3)** the *negation* of the proposition:
$p \rightarrow (\neg q)$

b) Consider the statement, "If Casey strikes out, then there is no joy in Mudville." Express in natural English **(1)** the *converse*, **(2)** the *contrapositive*, and **(3)** the *negation* of this statement.

10. Consider an ordinary poker deck of 52 playing cards. For how many cards in the deck is it true

a) that "This card is either a Heart or a Spade"?

b) that "This card is both a King and a Diamond"?

c) that "This card is either a King or a Diamond"?

d) that "This card is a King if and only if it is a Heart"?

e) that "If this card is a Queen, then it is a Diamond"?

f) that "If this card is an Ace, then it is a King"?