This is the homework for the week of September 7 to 11, covering Chapter 1, Sections 4 and 5. It is due in class on Wednesday, September 16. Don't forget to show your work!

1. Express the following statements in predicate logic. Try to express as much of the meaning as you can. When it is not clear, state the domain of discourse of your predicates.

a) Everyone is rich.

b) Everyone in Geneva is rich.

c) There is a city where everyone is rich.

d) There is a book that no one has read.

e) Someone in Geneva owns a brown dog.

f) Some pink elephants live in trees, but no pink elephants live in purple houses.

2. Let $F(p,t)$ stand for "you can fool person x at time t". Translate the following sentence into predicate logic: "You can fool some of the people all of the time, and you can fool all of the people some of the time, but you can't fool all of the people all of the time."

3. Find the negation of each of the following expressions. Simplify the answer, so that the operator $\neg$ is only applied to individual predicates.

a) $\forall x(P(x) \vee Q(x))$ **b)** $\forall s\big((\exists t\, L(s,t)) \vee (\exists t\, L(t,s)) \big)$

c) $\forall y\big(H(y) \rightarrow (\exists z\, K(y,z)) \big)$ **d)** $\exists x \exists y\big(R(x,y) \wedge \forall z(S(x,z) \vee T(y,z)) \big)$

4. Suppose that $S(p,m)$ means "p has seen m," where the domain of discourse for p is people, and the domain of discourse for m is movies. State the meaning of each of the following propositions as an unambiguous English sentence:

a) $\exists p\, \exists m(S(p,m))$ **b)** $\forall p\, \exists m(S(p,m))$ **c)** $\exists m\, \forall p(S(p,m))$

d) $\forall p\, \forall m(S(p,m))$ **e)** $\exists p\, \forall m(S(p,m))$ **f)** $\forall m\, \exists p(S(p,m))$

5. Use a *truth table* to decide whether the following argument is valid:

$$\begin{array}{l} p \rightarrow q \\ (\neg p) \rightarrow q \\ \hline \therefore\ q \end{array}$$

6. Give a formal proof for each of the following valid arguments, includings the justification for each step in the proof:

a)
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow (r \vee s) \\ \neg s \\ p \\ \hline \therefore\ r \end{array}$$

b)
$$\begin{array}{l} (p \wedge q) \rightarrow (r \vee s) \\ \neg r \\ p \rightarrow q \\ p \\ \hline \therefore\ s \end{array}$$

c)
$$\begin{array}{l} p \rightarrow r \\ (r \wedge s) \rightarrow t \\ q \rightarrow \neg t \\ s \\ q \\ \hline \therefore\ \neg p \end{array}$$

7. Translate each of the following informal English arguments into informal argument by assigning propositional variables to represent the basic statements used in the arguments. Then decide whether or not the argument is valid. If the argument is valid, give a formal proof. If it is not valid, explain why.

a) If Jack fails this course, then he won't graduate on time. If Jack doesn't study, then he will fail this course. Jack studies. So Jack will graduate on time.

b) In order to get a B.S. degree, you must pass a math class or a computer science class. If you don't understand algebra, you can't pass a math class. Mary has a B.S. degree, but Mary doesn't understand algebra. So Mary must have taken a computer science class.