*There is a test next Wednesday, September 23, covering Sections 1.1 through 1.7. Because of the test, this homework is due on **Monday**, September 21.*

*For the proofs that you are asked to give on this homework, you should give informal, but careful and complete, proofs of the kind that are typically given by mathematicians. In your proofs, you can use the following restuls from class and the book without re-proving them:*

- *The product of any two rational numbers is rational.*
- *The sum of any two rational numbers is rational.*
- *The number $\sqrt{2}$ is irrational.*

*You can also use basic facts from algebra.*

**1.** Each of the following statements is a potential theorem that is expressed somewhat informally, following the usual mathematical practice. Translate each statement into a statement of formal predicate logic, using quantifiers as appropriate. Introduce predicates and variables as necessary. You are **not** being asked to prove the statements in this problem, only to translate them into logic.

**a)** If $a$ and $b$ are odd integers, then $ab$ is also an odd integer.

**b)** Not every prime number is odd.

**c)** Suppose that $p$ is a prime number and that $n$ and $m$ are integers such that the product $nm$ is divisible by $p$. Then either $n$ is divisible by $p$ or $m$ is divisible by $p$.

**2.** Suppose that $a$, $b$, and $c$ are integers and that $a$ is divisible by $c$. Prove that the product $ab$ is also divisible by $c$.

**3.** Find a counterexample to disprove the following statement: Suppose $a$, $b$, and $c$ are integers and that the product $ab$ is divisble by $c$. Then either $a$ is divisble by $c$ or $b$ is divisible by $c$.

**4.** Prove or disprove:

**a)** If $x$ is an irrational number, then $x^2$ is also irrational.

**b)** If $x^2$ is an irrational number, then $x$ is also irrational.

**5.** Prove: If $x$ is any real number, then either $x$ or $\pi - x$ is irrational. [Note: $\pi$ is known to be an irrational number, and you can assume that that fact is already proved.]

**6.** Use a proof by contradiction to prove: If $x$ is an irrational number and $r$ is a non-zero rational number, then $xr$ is an irrational number.

**7.** Prove that for any integer $n$, the number $n^2 + n$ is even. (Consider a proof by cases, looking at the case where $n$ is even and the case where $n$ is odd.)