Mathematical theorems and proofs are more informal than the arguments and formal proofs that we studied previously, but they are still based on logic. A theorem has *hypotheses*, which are like premises, and a truly complete proof would consist of a series of statements where each statement is known to be true either because it is an hypothesis, an axiom, a definition, or a previously proved result, or because it follows logically from previous statements in the proof. This handout lists some possible approaches to proving statements with some common logical forms.

To Prove:	Try this:
$\forall x\, P(x)$	Let a be arbitrary (in the domain of discourse). Prove $P(a)$.
$\forall x\, \forall y\, P(x,y)$	Let a and b be arbitrary (in the domain of discourse). Prove $P(a,b)$.
$\exists x\, P(x)$	**Existence proof:** Exhibit a specific a (in the domain of discourse). Prove $P(a)$.
$\neg \forall x\, P(x)$	Prove $\exists x\, \big(\neg P(x)\big)$.
$\neg \exists x\, P(x)$	Prove $\forall x\, \big(\neg P(x)\big)$.
$p \to q$	Assume p. (That is, take it as an additional premise.) Prove q (based on the assumption that p is true).
$p \to q$	Prove the contrapositive, $\neg q \to \neg p$. That is, assume $\neg q$, and prove $\neg p$.
$p \leftrightarrow q$	Prove $p \to q$; and prove $q \to p$.
$p \leftrightarrow q$	Find a sequence of propositions $r_1, r_2, \ldots, r_k$ such that p iff r_1 iff r_2 iff $\cdots$ iff r_k iff q
$p \vee q$	Assume $\neg p$. Prove q.
p	**Proof by contradiction:** Assume $\neg p$. Prove some r, where r is a contradiction (that is, $r \equiv \mathbb{F}$).
p	**Proof by cases:** Consider cases $r_1, r_2, \ldots, r_k$ that exhaust all possibilities; that is, $r_1 \vee r_2 \vee \cdots \vee r_k$ is a tautology. (A common possibility is to use q and $\neg q$ for some propostion q.) Prove $r_1 \to p$, $r_2 \to p$, \ldots, and $r_k \to p$.
$\forall n\, \big(n \geq K \to P(n)\big)$	**Proof by Mathematical Induction.** (Here, the domain of discourse is integers.) Prove $P(K)$. Prove for all $n \geq K$, $P(n) \to P(n+1)$.

Some General Proof Moves:

- Apply an assumption/hypothesis/premise!

- Apply a definition!

- Apply a previously proved (or known) fact.