

This homework covers the reading for the first week of classes: Chapter 1, Sections 1 and 2 (but mostly Section 1). It is due in class on Friday, January 26. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit. You are encouraged to come in for help on doing the homework and on understanding the material, if you need it.

1. Draw truth tables to prove the following logical equivalences (and justify your answer by saying what it is about the table that proves equivalence):

a) $p \wedge (q \vee \neg p) \equiv p \wedge q$

b) $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

2. Now, prove the same logical equivalences using Boolean Algebra. That is, find a chain of logical equivalences leading from the left side of the equivalence to the right side, where a single definition or rule of Boolean algebra is applied in each step. For each step, state what definition or rule you are applying. (One rule that you will need is the fact that $a \rightarrow b \equiv (\neg a) \vee b$.)

a) $p \wedge (q \vee \neg p) \equiv p \wedge q$

b) $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

3. In each of the following pairs of propositions, one of the propositions is a tautology. Which one? (Justify your answer!)

a) $(p \vee q) \rightarrow (p \wedge q), \quad (p \wedge q) \rightarrow (p \vee q)$

b) $p \rightarrow (p \wedge q), \quad p \rightarrow (p \vee q)$

c) $p \rightarrow (p \rightarrow q), \quad p \rightarrow (q \rightarrow p)$

4. Convert each of the following English statements into propositional logic. You should introduce symbols (such as p, q, d, f , etc.) to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for.

a) *Victor is smart but not lucky.*

b) *If Cassandra is smart and lucky, she will be rich or famous.*

c) *Achilles is brave and famous or long-lived.* (This statement is **ambiguous**; you should give **two** possible translations and explain the difference.)

5. Express the negation of each of the following sentences in natural, unambiguous English.

a) *The answer is less than 17 or is greater than or equal to 42.*

b) *Victor is smart but not lucky.*

c) *If Achilles lives to be 100, he will be famous.*

(over)

6. a) Find (1) the *converse*, (2) the *contrapositive*, and (3) the *negation* of the proposition:
 $p \rightarrow (\neg q)$
- b) Consider the statement, “*If Casey strikes out, then there is no joy in Mudville.*” Express in natural English (1) the *converse*, (2) the *contrapositive*, and (3) the *negation* of this statement.
7. Consider an ordinary poker deck of 52 playing cards. Using the definitions of the logical operators, determine the number of cards in the deck for which it is true that
- a) “This card is both an Ace and a Spade”?
 - b) “This card is either an Ace or a Spade”?
 - c) “This card is an Ace but is not a Spade”?
 - d) “If this card is a Ace, then it is a Spade”?
 - e) “If this card is an Ace, then it is a King”?