This homework covers the reading for the second week of classes: Chapter 1, Sections 3, 4, and 5. It is due in class on Friday, February 2. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.

1. Express the following statements in predicate logic. Try to express as much of the meaning as you can. Give the meaning of each predicate that you use. When it is not clear, state the domain of discourse of your predicates.

a) All cows are brown.

b) Every cow in Texas is brown.

c) There is a state where every cow is brown.

d) There is a book that no one has read.

e) Everyone who owns a pink unicorn is lucky.

2. Find the negation of each of the following expressions. Simplify the answer, so that the operator $\neg$ is only applied to individual predicates. (Show your work by writing a chain of logical equivalences, starting from the negation of the given statement.)

a) $\forall x(P(x) \vee Q(x))$

b) $\forall s\big((\exists t\, L(s,t)) \vee (\exists t\, L(t,s)) \big)$

c) $\forall y\big(H(y) \to (\exists z\, K(y,z)) \big)$

d) $\exists x \exists y\big(R(x,y) \wedge \forall z(S(x,z) \vee T(y,z)) \big)$

3. Translate the following English sentence directly into predicate logic: "Not everyone who lives in Geneva is a student." Then simplify the resulting expression by applying the rules for negation. And finally, translate the result back into English.

4. a) Use a *truth table* to show that the following argument is valid:

$$\begin{array}{l} p \to q \\ (\neg p) \to q \\ \hline \therefore\ q \end{array}$$

b) Explain in English why it *makes sense* that this argument is valid. (What does the argument *mean*?)

5. Give a formal proof for each of the following valid arguments, including the justification for each step in the proof:

a)
$$\begin{array}{l} p \to q \\ q \to (r \vee s) \\ \neg s \\ p \\ \hline \therefore\ r \end{array}$$

b)
$$\begin{array}{l} (p \wedge q) \to (r \vee s) \\ \neg r \\ p \to q \\ p \\ \hline \therefore\ s \end{array}$$

c)
$$\begin{array}{l} p \to r \\ (r \wedge s) \to t \\ q \to \neg t \\ s \\ q \\ \hline \therefore\ \neg p \end{array}$$

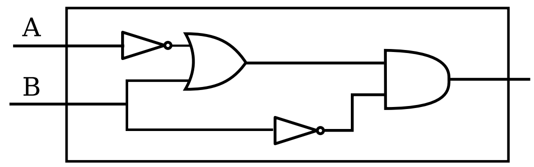
6. Translate each of the following informal English arguments into a formal argument by assigning propositional variables to represent the basic statements used in the argument. Then decide whether or not the argument is valid. If the argument is valid, give a formal proof. If it is not valid, explain why.

a) If Jack fails this course, then he won't graduate on time. If Jack doesn't study, then he will fail this course. Jack studies. So Jack will graduate on time.

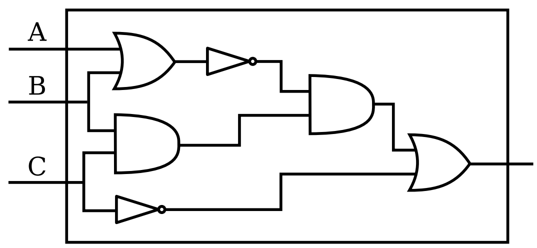
b) If Pete is at the party, then so is Quentin. If Quentin and Roger are both at the party, then so is Tom. Tom is not at the party, but Roger is. So, Pete is not at the party.

7. Find the Boolean expression that gives the output of each circuit as a function of its inputs. (Show your work by redrawing the circuit and labeling the output of each logic gate.)

a)



b)



8. Two logic circuits are equivalent if they have the same output for any set of possible inputs. Circuit **a)** in the previous problem is equivalent to a simpler circuit. Use Boolean algebra to find such a circuit. (Find one that uses the minimum possible number of gates.)

9. Draw the logic circuit that computes each of the following propositions.

a) $(A \wedge \neg B) \vee (B \wedge C)$ **b)** $(A \vee B \vee C) \wedge \neg(A \wedge B \wedge C)$