1. Use a proof by induction to show that the following method correctly finds the sum of array elements `A[0]`, `A[1]`, ..., `A[N-1]` for all $N \geq 1$.

```
int recursive_sum( int[] A, int N ) {
    if ( N == 1 )
        return A[0];
    else {
        return A[N-1] + recursive_sum( A, N-1 );
    }
}
```

2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$; $B = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$; $C = \{n \in \mathbb{Z} \mid -5 \leq n \leq 5\}$. Find the following sets. (For this exercise, you do **not** need to justify your answers.)

   a) $A \cup B$ **b)** $A \cap B$ **c)** $A \smallsetminus B$ **d)** $B \smallsetminus A$

   e) $A \cap C$ **f)** $\mathbb{N} \cup C$ **g)** $\mathbb{N} \smallsetminus C$ **h)** $\mathbb{Z} \smallsetminus A$

(Recall that $\mathbb{N}$ is the set of natural numbers and $\mathbb{Z}$ is the set of integers.)

3. Let S be the set $S = \{\emptyset, s, \{s\}\}$. Write out the power set, $\mathcal{P}(S)$. (You do not have to justify your answer.)

4. True or false: For any sets A and B, if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. Justify your answer.

5. Prove that for any sets A and B, $A \cap B \subseteq A$, using the definitions of $\cap$ and $\subseteq$.

6. (Exercise 2.1.7 from the textbook.) In the English sentence, "She likes men who are tall, dark, and handsome," does she like an intersection or a union of sets of men? How about in the sentence, "She likes men who are tall, men who are dark, and men who are handsome"? Explain.

7. Let k, n, and m be values of type *int* given as hexadecimal numbers in Java as

           `k = 0xFF00`   `m = 0xAB24`   `n = 0x77`

Find the values of the following Java expressions, writing the answers as hexadecimal numbers.

   a) `k >> 4`   **b)** `(k & m) | n`   **c)** `(m << 16)|k|n`   **d)** `m & n`

8. Consider the two 16-bit integers n and m shown below. Compute the three 16-bit integers ~n, and n & m, and $n \mid m$. What subset of $\{0, 1, 2, \ldots, 15\}$ does each of the integers n, m, ~n, n & m, and $n \mid m$ correspond to? (Write out each set in full.)

$$n = 1001\ 1101\ 1000\ 0101$$
$$m = 0101\ 1001\ 1100\ 0111$$

9. What is computed by the following method? (Hint: Write N in binary.) Explain your answer.

```
int countSomething( int N ) {
    int ct = 0;
    for (int i = 0; i <= 31; i++) {
        if ( (N & 1) == 1 ) {
            ct++;
        }
        N = N >> 1;
    }
    return ct;
}
```