This homework, on Sections 3.1 and 3.2, is due on Monday, March 5. On Friday of this week, the class will meet in Rosenberg 009 for some work with regular expressions. The completed work will be due the following Friday.

1. Suppose that L and M are languages over the alphabet $\Sigma = \{a, b\}$, defined as follows:

$$L = \{a, aa\} \qquad M = \{w \in \Sigma^* \mid w \text{ ends with } b\}$$

Identify the following languages. (Write the language out as a set, or give a clear English description of the language; explain your reasoning if the answer is not obvious.)

a) $L \cap M$ **b)** $L \cup M$ **c)** L^3 **d)** L^* **e)** ML
f) LM **g)** M^* **h)** $\overline{M}$ **i)** M^R **j)** $M^R M$

2. Give an English description of the language generated by each of the following regular expressions over the alphabet $\{a, b\}$, or write out the answer as a set:

a) bab^* **b)** $b(ab)^*$ **c)** $(bab)^*$

d) $(a|b)^*bbb(a|b)^*$ **e)** $a^*ba^*ba^*ba^*$ **f)** $a^*(b|\varepsilon)a^*(b|\varepsilon)a^*(b|\varepsilon)a^*$

3. Find a regular expression that generates each of the following languages over the alphabet $\{0, 1\}$, and explain in words how your regular expression works.

a) $\{w \in \Sigma^* \mid |w| \geq 2$ and w starts and ends with the same symbol $\}$

b) $\{w \in \Sigma^* \mid$ every 1 in w is immediately followed by a $0\}$

c) $\{w \in \Sigma^* \mid |w|$ is an odd number$\}$

4. Find a regular expression that generates each of the following languages over the alphabet $\Sigma = \{a, b\}$, and explain in words how your regular expression works.

a) $\{w \in \Sigma^* \mid$ the number of a's in w is an odd number $\}$

b) $\{w \in \Sigma^* \mid w$ contains the string $ab\}$

c) $\{w \in \Sigma^* \mid w$ does **not** contain the string $ab\}$

5. Explain why every *finite* langauge is *regular*.

6. Let L be a non-empty language. Show that $L \subseteq L^2$ if and only if $\varepsilon \in L$. That is, prove **both**

a) Let L is a non-empty language. If $\varepsilon \in L$, then $L \subseteq L^2$.

b) Let L is a non-empty language. If $L \subseteq L^2$, then $\varepsilon \in L$. (Hint: Consider a string x in L that has minimal length.)