CS 229, Spring 2018 — Sample Answers To Final Study Guide Questions

①

p	q	$\neg p$	$p \to q$	$(\neg p) \to q$	$(p \to q) \wedge ((\neg p) \to q)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Since these two columns are the same,

$$q \equiv (p \to q) \wedge ((\neg p) \to q)$$

② a) $\neg(p \vee q \vee (\neg r)) \equiv (\neg p) \wedge (\neg q) \wedge (\neg(\neg r))$

$\equiv (\neg p) \wedge (\neg q) \wedge r$

b) $\neg(\forall x\, \exists y\, (L(x,y) \wedge \forall z\, (\neg L(x,z))))$

$\equiv \exists x\, \forall y\, \neg(L(x,y) \wedge \forall z\, (\neg L(x,z)))$

$\equiv \exists x\, \forall y\, (\neg L(x,y) \vee \neg \forall z (\neg L(x,z)))$

$\equiv \exists x\, \forall y\, (\neg L(x,y) \vee \exists z (L(x,z)))$

c) $\neg(\forall x (P(x) \to \exists y (R(y) \wedge Q(x,y))))$

$\equiv \exists x (\neg (P(x) \to \exists y (R(y) \wedge Q(x,y)))$

$\equiv \exists x\, (P(x) \wedge \neg \exists y (R(y)) \wedge Q(x,y)))$

$\equiv \exists x\, (P(x) \wedge \forall y\, (\neg (R(y) \wedge Q(x,y))))$

$\equiv \exists x\, (P(x) \wedge \forall y ((\neg R(y)) \vee (\neg Q(x,y))))$

(3) a) I do well in CS 229, but not everyone thinks I'm cool.

b) Let $P(x) \equiv x$ is a planet, $H(x) \equiv x$ is habital

$M(x) \equiv x$ is a moon, $O(x,y) \equiv x$ orbits y.

$$\forall x \left((P(x) \land H(x)) \rightarrow \exists m (M(m) \land O(m,x)) \right)$$

c) $\forall x \exists y F(x,y)$: "For every person, there is something that that person fears."

$\exists y \forall x F(x,y)$: "There is something such that every person fears that thing."

(4) Let $j \equiv$ "Jack has a day off"

$r \equiv$ "It's raining"

$s \equiv$ "The sun is shining.

Argument:

$$\begin{array}{l} j \rightarrow r \\ s \rightarrow \neg r \\ j \\ \hline \therefore \neg s \end{array}$$

Proof:

1. $j \rightarrow r$ premise
2. j premise
3. r from 1, 2 by Modus Ponens
4. $s \rightarrow \neg r$ premise
5. $\neg(\neg r)$ from 3, double negation
6. $\neg s$ from 4, 5 by Modus Tollens

(5) Suppose A, B, C are sets, and $A \subseteq B$ and $A \subseteq C$. WWTS $A \subseteq B \cap C$. Let a be an arbitrary element of A. Since $A \subseteq B$ and $a \in A$, then $a \in B$ by

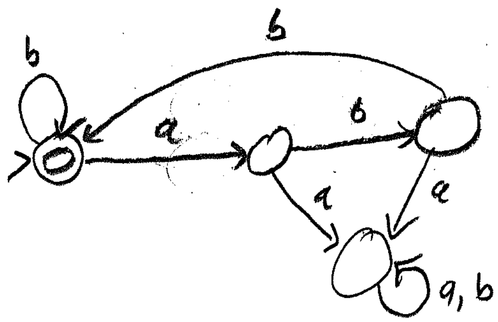
definition of $\subseteq$. Since $A \subseteq C$ and $a \in A$, Then $a \in C$ by definition of $\subseteq$. Since $a \in B$ and $a \in C$, Then $a \in B \cap C$ by definition of $\cap$. We have shown That for any $a \in A$, $a \in B \cap C$. By definition of $\subseteq$, This shows $A \subseteq B \cap C$. QED

(6) Suppose A and B are sets and $A \cup B$ is uncountable. WWTS at least one of A and B is uncountable. Suppose, FSOC, That both A and B are countable. We know That The union of Two countable sets is countable; that is, $A \cup B$ is countable. But This contradicts The fact That $A \cup B$ is uncountable. This contradiction proves That A and B cannot both be countable. So, at least one of Them is uncountable.

(7) $\{ w \in \{a,b\}^* \mid |w|$ is not a multiple of $3 \}$.

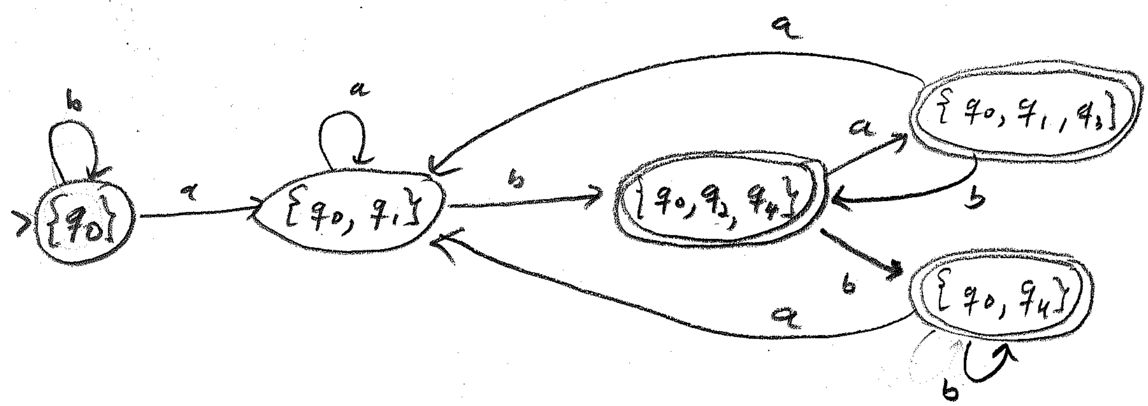
[$((a|b)(a|b)(a|b))^*$ generates all strings whose lengths are mutiples of 3. $(a|b)(\varepsilon|a|b)$ adds either one or Two more symbols To The string, giving a string whose length is of The form $3n+1$ or $3n+2$.]

(8)



[After reading an a, Two b's must be read in a row To get back To The accepting state.]

(9)



(10) The language $\{a^n b^n a^n b^n \mid n \in \mathbb{N}\}$ is <u>NOT</u> context-free

$\{a^n b^m a^m b^n \mid n, m \in \mathbb{N}\}$

$S \to aSb$
$S \to T$
$T \to bTa$
$T \to \varepsilon$

$\{a^n b a^m b a^{n+m} \mid n, m \in \mathbb{N}\}$

$S \to aSa$
$S \to bT$
$T \to aTa$
$T \to b$

(11)
$S \to TbZ$
$T \to aTD$
$T \to X$
These rules can produce strings of the form $a^n X D^n b Z$

$Db \to bbD$ } Each D will double the number of b's as it moves from left to right

$DZ \to Z$ } The D's disappear as they reach the Z at the end.

$Xb \to bX$ } X can move right past all the b's.

$XZ \to \varepsilon$ } X and Z disappear when X reaches the end.

This grammar produces the language

$\{a^n b^{2^n} \mid n \in \mathbb{N}\}$

If there are n a's, there will also be n D's, so the number of b's will be doubled n times, giving b^{2^n}.

(12) $S \to XTaY$
$T \to DT$
$T \to \varepsilon$
} produces XD^naY, for any n.

$Da \to aaaD$ } each D triples the number of a's, giving $Xa^{3^n}D^nY$.

$DY \to Y$ } D's disappear when they hit the Y.

$Xa \to aX$
$XY \to a$
} X moves right to meet the Y. They are converted into one additional a, giving a^{3^n+1}

(13) a) The machine simply moves right one cell, then left one cell, and halts with the tape still empty.

b) The output is b.

[The machine goes through this sequence of moves:

$ab\overset{q_0}{a} \Rightarrow a\overset{q_0}{b}\$ \Rightarrow a\$\overset{q_1}{\$} \Rightarrow a\$\$\overset{q_1}{\#} \Rightarrow a\$\overset{q_2}{\$}b$

$\Rightarrow a\overset{q_0}{\$}\$b \Rightarrow \overset{q_0}{a}\$\$b \Rightarrow \overset{q_0}{\#}\$\$\$b \Rightarrow \overset{q_3}{\$}\$\$b$

$\Rightarrow \overset{q_3}{a}\$\$b \Rightarrow \#\#\overset{q_3}{\$}b \Rightarrow \#\#\#\overset{q_3}{b} \Rightarrow b\overset{q_3}{\#} \Rightarrow \overset{h}{b}$]

c) The machine converts a string of a's and b's into a string containing only b's, where the number of b's is equal to the number of b's in the original string

(Answers to essay questions are omitted.)