CS229, Spring 2017 Sample Answers for Test 1 Study Guide questions

1. a) Two propositions are logically equivalent if they have the same truth value for all possible combinations of values for the propositional variables that they contain.

   b) $A \subseteq B$ means $\forall x\,(x \in A \to x \in B)$. (A set A is a subset of a set B if every element of A is also an element of B.)

   c) A set is countably infinite if there is a one-to-one correspondence from $\mathbb{N}$ to the set.

2. a) If the Greenland ice sheet melts, then global temperatures rise by two degrees.

   b) If the Greenland ice sheet does not melt, then global temperatures don't rise by two degrees.

   c) Global temperatures rise by two degrees, but the Greenland ice sheets don't melt.

3.

p	q	r	$p \wedge q$	$(p \wedge q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

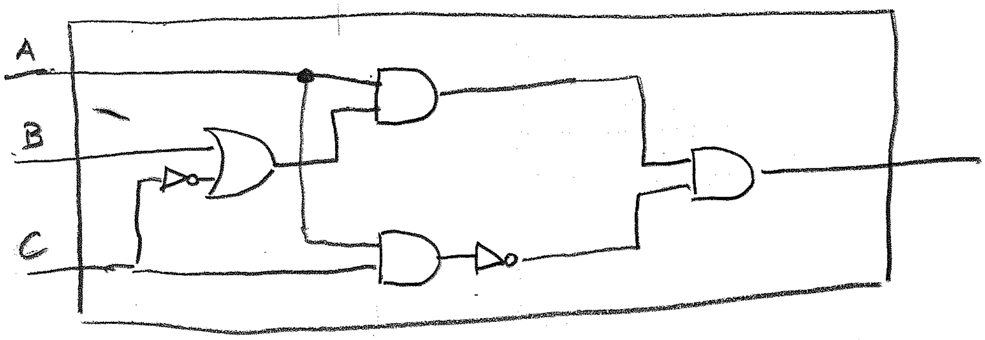
Since these columns are the same, the propositions are logically equivalent.

④ a) $\exists x\,(M(x) \wedge I(x))$

b) $\forall x\,(M(x) \rightarrow D(x))$

c) $\forall x\,((M(x) \wedge D(x)) \rightarrow I(x))$

⑤



⑥ $\neg(\exists x\,(P(x) \wedge \forall y\,(Q(x,y) \vee R(x,y))))$

$\equiv \forall x\,[\neg\,(P(x) \wedge \forall y\,(Q(x,y) \vee R(x,y)))]$

$\equiv \forall x\,[(\neg P(x)) \vee \exists y\,(\neg\,(Q(x,y) \vee R(x,y))]$

$\equiv \forall x\,[(\neg P(x)) \vee \exists y\,((\neg Q(x,y)) \wedge (\neg R(x,y)))]$

⑦ 1. $s \rightarrow t$ premise

2. $\neg t$ premise

3. $\neg s$ from 1, 2 by Modus Tollens

4. q premise

5. $q \wedge (\neg s)$ from 3, 4

6. $(q \wedge (\neg s)) \rightarrow p$ premise

7. p from 5, 6 by Modus Ponens

(8) In $\exists y \forall x L(x,y)$, there is one particular y, say $y = y_0$, such that $L(x, y_0)$ is true for every x. In $\forall x \exists y L(x,y)$, there can be a different y for each x for which $L(x,y)$ is True. For example, if $L(x,y)$ means "x likes y", then $\exists y \forall x L(x,y)$ means that "there is someone who is liked by everyone", whereas $\forall x \exists y L(x,y)$ means "Every person has someone whom they like."

(9) Let x be an integer, and suppose x^2 is odd. WWTS x is odd. Suppose x is <u>NOT</u> odd. Then n is even, so n can be written $x = 2k$ for some integer k. But then $x^2 = (2k)^2 = 2\cdot(2k^2)$, which shows x^2 is even, contradicting the fact that x^2 is odd.

(10) Suppose a, b, and c are integers and a is divisible by c and b is divisible by c. Then $a = ck$ for some integer k, and $b = cl$ for some integer l. So $a+b = ck + cl = c(k+l)$. Since $k+l$ is an integer, this shows $a+b$ is divisible by c.

(11) a) $A \cap B = \{2, 3\}$ b) $A \setminus B = \{5, 7, 11, 13\}$

c) $C \times B = \{ (a,1), (a,2), (a,3), (b,1), (b,2), (b,3) \}$

d) $P(C) = \{ \emptyset, \{a\}, \{b\}, \{a, b\} \}$

(12) a) subset b) element c) both

(13) $n \,\&\, 0xFFF000 = 0xABCDEF \,\&\, 0xFFF000$

$= 0xABC000$

$m \,\&\, 0xFFF = 0x123456 \,\&\, 0x000FFF = 0x000456$

So $(n \,\&\, 0xFFF000) \,|\, (m \,\&\, 0xFFF) = \underline{0xABC456}$

(14) A universal set is only defined for a particular context. It contains all the entities that are under consideration in that context. A universal set, U, can be used to define set complement, $\overline{A} = \{x \in U \mid x \notin A\}$. For example, if we are only talking about integers, the universal set might be $\mathbb{Z}$, and we would have $\overline{\mathbb{N}} = \{\ldots, -3, -2, -1\}$.

(15) The principle of mathematical induction says the following: Suppose $P(n)$ is a proposition, where the domain of discourse is natural numbers. If we prove $P(M)$ for some number M [the "base case"], and we prove that for any $k \geq M$, $P(k) \rightarrow P(k+1)$, then $P(n)$ is true for all $n \geq M$. This is obviously true since $P(M)$ and $P(M) \rightarrow P(M+1)$ together prove $P(M+1)$. Then $P(M+1)$ and $P(M+1) \rightarrow P(M+2)$ together prove $P(M+2)$, and so on forever.