The first test for this course will be given in class on Wednesday, February 21. It covers all of the material that we have done in Chapters 1 and 2 (with somewhat more emphasis on Chapter 1).

The test will include some "short essay" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework. You can expect to do a few simple proofs, including at least one formal proof of the validity of an argument and at least one more informal mathematical proof. There might be a simple proof by induction.

Here are some terms and ideas that you should be familiar with for the test:

translations from logic to English, and from English to logic

proposition; propositional logic

the logical operators "and" ($\wedge$), "or" ($\vee$), and "not" ($\neg$)

some information is lost when translating English to logic (for example, "but" vs. "and")

truth table

logical equivalence ($\equiv$)

the conditional or "implies" operator ($\rightarrow$)

definition of $p \rightarrow q$ as $(\neg p) \vee q$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

tautology

Boolean algebra

some basic laws of Boolean algebra (double negation, De Morgan's, commutative, etc.)

logic circuits and logic gates

making a circuit to compute the value of a compound proposition

finding the proposition whose value is computed by a circuit

converse of an implication ($p \rightarrow q$ has converse $q \rightarrow p$)

contrapositive of an implication ($p \rightarrow q$ has contrapositive $(\neg q) \rightarrow (\neg p)$)

an implication is logically equivalent to its contrapositive

predicates; predicate logic

one-place predicate, two-place predicate, etc.

domain of discourse

quantifiers, "for all" ($\forall$) and "there exists" ($\exists$)

negation of a statement that uses quantifiers, such as $\neg \exists x\,(P(x)) \equiv \forall x\,(\neg P(x))$

arguments, valid arguments, and deduction

premises and conclusion of an argument

formal proof of the validity of an argument

how to show that an argument is invalid

translating arguments from English into logic

Modus Ponens and Modus Tollens

mathematical proof

various "moves" that can be used in a proof
existence proof
counterexample
proof by contradiction
rational number (real number that can be expressed as a quotient of integers, $\frac{a}{b}$)
irrational number (real number that is not rational such as π or $\sqrt{2}$)
divisibility (for integers n and m, n is divisible by m if $n = km$ for some integer k)
prime number (greater than 1, and cannot be factored into smaller integers)
proof by mathematical induction
summation notation, for example: $\sum_{k=1}^{n} a_k$
sets
set notations: $\{a, b, c\}$, $\{1, 2, 3, \dots\}$, $\{x \mid P(x)\}$, $\{x \in A \mid P(x)\}$
the empty set, $\emptyset$ or $\{\,\}$
equality of sets: $A = B$ if and only if they contain the same elements
element of a set: $a \in A$
subset: $A \subseteq B$
union, intersection, and set difference: $A \cup B$, $A \cap B$, $A \smallsetminus B$
definition of set operations in terms of logical operators
disjoint sets ($A \cap B = \emptyset$)
power set of a set: $\mathcal{P}(A)$
universal set
complement of a set (in a universal set): $\overline{A}$
DeMorgan's Laws for sets: $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$
bitwise operations in Java: `&`, `|`, `~`
correspondence between n-bit binary numbers subsets of $\{0, 1, 2, \dots, n-1\}$
`&`, `|`, and `~` as set operations (intersection, union, complement)
the shift operators `<<`, `>>`, and `>>>`
hexadecimal numbers
ordered pair: (a, b)
cross product of sets: $A \times B$
function $f\colon A \to B$
one-to-one correspondence
cardinality of a finite set: $|A|$
finite set (in one-to one correspondence with one of the sets $N_0, N_1, N_2, \dots$)
infinite set (not finite)
countably infinite set (in one-to-one correspondence with $\mathbb{N}$)
uncountable set (infinte but not countably infinite)
examples of countably infinite and uncountably infinite sets
for any set A, there is no one-to-one correspondence between A and $\mathcal{P}(A)$
the set $\mathbb{R}$ of real numbers is uncountable
diagonalization argument

Here are some problems from tests given in previous courses:

*Note that this is **more** than a complete test, more like 1.5 tests!*

1. Define the following terms, as they relate to this course:

a) *logical equivalence* of propositions

b) *subset*

c) *countably infinite set*

2. Let p be the following proposition: "If global temperatures rise by two degrees, then the Greenland ice sheet melts."

a) State the *converse* of p (in natural English)

b) State the *contrapositive* of p (in natural English)

c) State the *negation* of p (in natural English):

3. Use a truth table to show that the proposition $(p \wedge q) \rightarrow r$ is logically equivalent to the proposition $p \rightarrow (q \rightarrow r)$. (Don't forget to say what it is about the truth table that demonstrates the equivalence.)

4. Suppose that $M(x)$ means "x is a math class"; $D(x)$ means "x is difficult"; and $I(x)$ means "x is interesting". The domain of discourse is "all classes." Using only these predicates, translate each of the following statements into predicate logic:

a) There is an interesting math class.

b) All math classes are difficult.

c) All difficult math classes are interesting.

5. Draw a logic circuit that computes the the following logical expression:

$$(A \wedge (B \vee (\neg C))) \wedge (\neg(C \wedge A))$$

6. Simplify the following, so that in the end the $\neg$ operator is applied only to individual predicates.

$$\neg(\exists x \left(P(x) \wedge \forall y \,(Q(x,y) \vee R(x,y)\right)$$

7. Give a *formal proof* that the following argument is valid. Don't forget to give a reason for each step in the proof.

$(q \wedge (\neg s)) \rightarrow p$

$s \rightarrow t$

$\neg t$

q

$\therefore p$

8. Consider the proposition $\forall x\, \exists y\, L(x, y)$ and the proposition $\exists y\, \forall x\, L(x, y)$. Carefully explain the difference in the meanings of these two propositions. It will be helpful to use some specific predicate L as an example.

9. Give a *proof by contradiction* of the following statement: If x^2 is an odd integer, then x is also odd.

10. Prove that for any integers a, b, and c, if a is divisible by c and b is divisible by c, then the sum $a + b$ is also divisible by c.

11. Let A, B, and C be the sets:

$$A = \{2, 3, 5, 7, 11, 13\} \qquad B = \{1, 2, 3\} \qquad C = \{a, b\}$$

Find the following sets:

a) $A \cap B =$ ______________________________

b) $A \smallsetminus B =$ ______________________________

c) $C \times B =$ ______________________________

d) $\mathcal{P}(C) =$ ______________________________

12. Let X be the set $X = \big\{\, a,\ \{b\},\ \{a, b\},\ \{a, \{b\}\},\ \{a, \{a\}\} \,\big\}$.

a) Is the set $\{a\}$ an *element* of X or a *subset* of X or neither or both? Why?

b) Is the set $\{a, b\}$ an *element* of X or a *subset* of X or neither or both? Why?

c) Is the set $\{a, \{b\}\}$ an *element* of X or a *subset* of X or neither or both? Why?

13. Suppose that n and m are variables of type *int* in a Java program, with values $n =$ `0xABCDEF` and $m =$ `0x123456`. What is the value of the following expression? (Explain your answer!)

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(n & 0xFFF000) | (m & 0xFFF)
```

14. What is meant by a *universal set*? Where are universal sets used, and why are they necessary. Give an example that uses a universal set.

15. Discuss the principle of mathematical induction. (What does it say? Why is it true, intuitively? What is meant by the base case and the inductive case of an induction?)