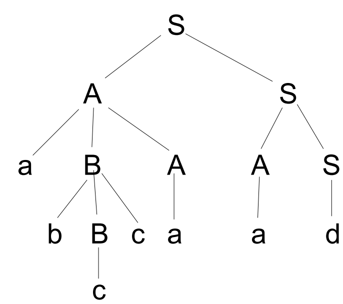
*The first two problems on this handout cover material for which you have not been assigned homework problems. The remaining problems are from an old test from a previous version of this course. This will give you some idea about they types of problems that might appear on our test, but remember that it is* ***not*** *meant as a comprehensive review of everything that you need to know for that test! You will also want to review definitions and old homework assignments on Chapters 3 and 4. The assignments and my answers can be found on the course web site. By Monday, I will post sample answers to the problems on this handout.*

**1.** Consider the parse tree shown at the right. The parse tree was created from a context-free grammar, $G$.




**a)** Find *five production rules* that must be part of the grammar $G$, given that the parse tree was constructed using the grammar $G$.

**b)** Find the *left derivation* corresponding to the parse tree.

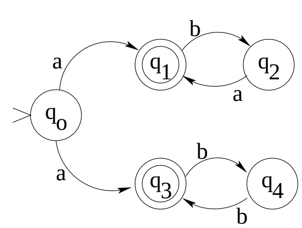
**2.** Consider the following context-free grammars:

$$\begin{aligned} S &\longrightarrow TR \\ T &\longrightarrow aTb \\ T &\longrightarrow aT \\ T &\longrightarrow a \\ R &\longrightarrow aRb \\ R &\longrightarrow bR \\ R &\longrightarrow b \end{aligned} \qquad\qquad \begin{aligned} S &\longrightarrow SS \\ S &\longrightarrow T \\ T &\longrightarrow aTb \\ T &\longrightarrow \varepsilon \end{aligned}$$

**a)** For the grammar on the **left**, find a *left derivation* and the corresponding *parse tree* for the string $aaabbabbb$.

**b)** For the grammar on the **right**, find **two** *left derivations* and the two corresponding *parse trees* for the string $abaabbab$.

**3.** Consider the following Nondeterministic Finite Automaton:




**a)** Give a regular expression for the language accepted by this NFA.

**b)** Apply the NFA-to-DFA conversion algorithm to construct a DFA that accepts the same language as this NFA.

**4.** Consider the regular language $L = \{w \in \{a,b\}^* \mid w \text{ contains the substring } abaab\}$. Draw a DFA that accepts exactly this language.

**5.** Give a regular expression that generates each language (no explanation necessary):

$$L_1 = \{w \in \{a,b\}^* \mid w \text{ contains } \textbf{at least 2 } a\text{'s}\}$$
$$L_1 = \{w \in \{a,b\}^* \mid w \text{ contains } \textbf{exactly } 2\ a\text{'s}\}$$

**6.** Draw an NFA that accepts the language over the alphabet $\Sigma = \{a,b,c\}$ that is generated by the regular expression $(a|b)^*cc^*(a|b)^*$. You do **not** have to use a specific regular-expression-to-NFA conversion algorithm; any NFA that works will do.

**7.** Let $L$ be a language over some alphabet $\Sigma$. Suppose that $\varepsilon \in L$. Show that $L \subseteq L^2$.

**8.** What does it mean that $x^R = x$ for a string $x$? Let $\Sigma = \{a,b,c\}$. Give several examples of strings, $x$, over the alphabet $\Sigma$ that have the property that $x^R = x$, and explain in English what this property means.

**9.** Give a Context-Free Grammar for the language $L = \{a^n b^m c\, b^m a^n \mid n \in \mathbb{N},\ m \in \mathbb{N}\}$, and briefly explain how your grammar works.

**10.** ("$a^n b^m$ variations") Match each language with the grammar that generates it. Enter the number for the correct grammer. No explanation is necessary.

| | | |
|---|---|---|
| **a)** $\{a^n b^m \mid n \neq m\}$ Grammar # ____ | **b)** $\{a^n b^m \mid n \geq m\}$ Grammar # ____ | **c)** $\{a^n b^m \mid n > 2m\}$ Grammar # ____ |
| **d)** $\{a^n b^m \mid n > 0\}$ Grammar # ____ | **e)** $\{a^n b^m \mid n > m > 0\}$ Grammar # ____ | **f)** $\{a^n b^m \mid n = 2m+1\}$ Grammar # ____ |

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**1)**
$S \longrightarrow aSB$
$S \longrightarrow \varepsilon$
$B \longrightarrow b$
$B \longrightarrow \varepsilon$

**2)**
$S \longrightarrow aS$
$S \longrightarrow Sb$
$S \longrightarrow a$

**3)**
$S \longrightarrow aaSb$
$S \longrightarrow aT$
$T \longrightarrow aT$
$T \longrightarrow \varepsilon$

**4)**
$S \longrightarrow aaTb$
$T \longrightarrow aTb$
$T \longrightarrow aT$
$T \longrightarrow \varepsilon$

**5)**
$S \longrightarrow aSb$
$S \longrightarrow T$
$S \longrightarrow R$
$T \longrightarrow aT$
$R \longrightarrow Rb$
$T \longrightarrow a$
$R \longrightarrow b$

**6)**
$S \longrightarrow aaSb$
$S \longrightarrow a$

**11.** Let $G = (V, \Sigma, P, S)$ be a Context-Free Grammar. $L(G)$ is the language generated by $G$. How is $L(G)$ defined? That is, exactly what does it mean for a string, $w$, to be in $L(G)$? (Mention $\Sigma$, $P$, and $S$ in your answer.)

**12.** NFA stands for "Nondeterministic Finite Automaton." Discuss how an NFA works and what it means to say that it is "nondeterministic." (What does an NFA do as it reads an input string? What does it mean to say that the NFA accepts the string?)