CS 229, Spring 2018 Some sample answers to Test 2 sample problems

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① a)

$S \to AS$

$A \to aBA$

$B \to bBc$

$B \to c$

$A \to a$

$S \to d$

b) $S \Rightarrow \underline{A}S$

$\Rightarrow a\underline{B}AS$

$\Rightarrow ab\underline{B}cAS$

$\Rightarrow abcc\underline{A}S$

$\Rightarrow abcca\underline{S} \Rightarrow abcca\underline{A}S$

$\Rightarrow abccaa\underline{S} \Rightarrow abccaad$

② a) $S \Rightarrow \underline{T}R$

$\Rightarrow a\underline{T}bR$

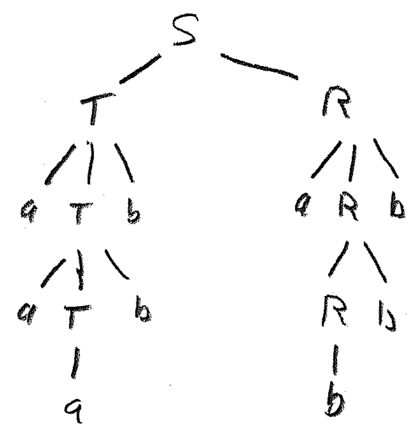
$\Rightarrow aa\underline{T}bbR$

$\Rightarrow aaabb\underline{R}$

$\Rightarrow aaabba\underline{R}b$

$\Rightarrow aaabbab\underline{R}b$

$\Rightarrow aaabbabbb$



b) $S \Rightarrow \underline{S}S$

$\Rightarrow \underline{S}SS \Rightarrow \underline{T}SS$

$\Rightarrow a\underline{T}bSS$

$\Rightarrow ab\underline{S}S$

$\Rightarrow ab\underline{T}S$

$\Rightarrow aba\underline{T}bS$

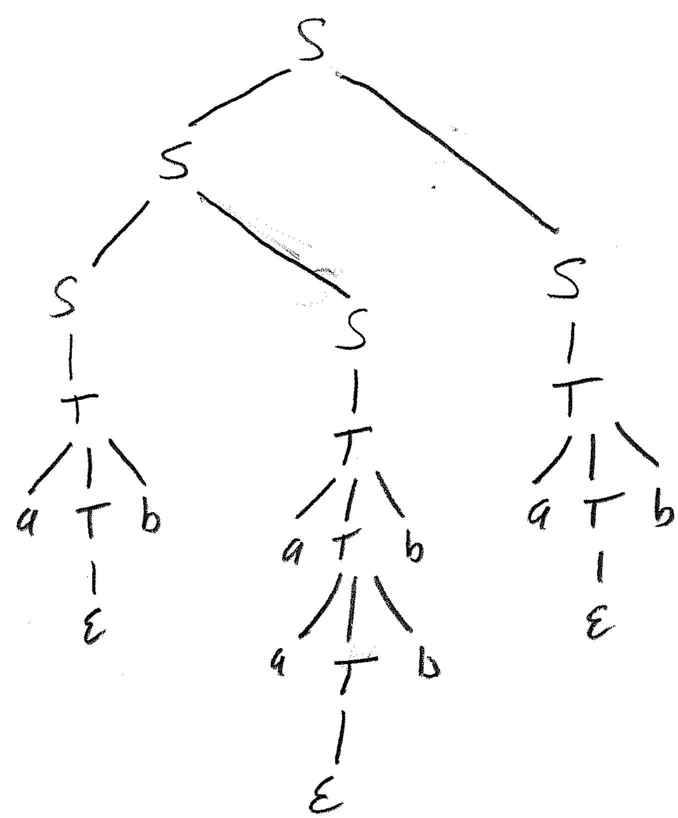
$\Rightarrow abaa\underline{T}bbS$

$\Rightarrow abaabb\underline{S}$

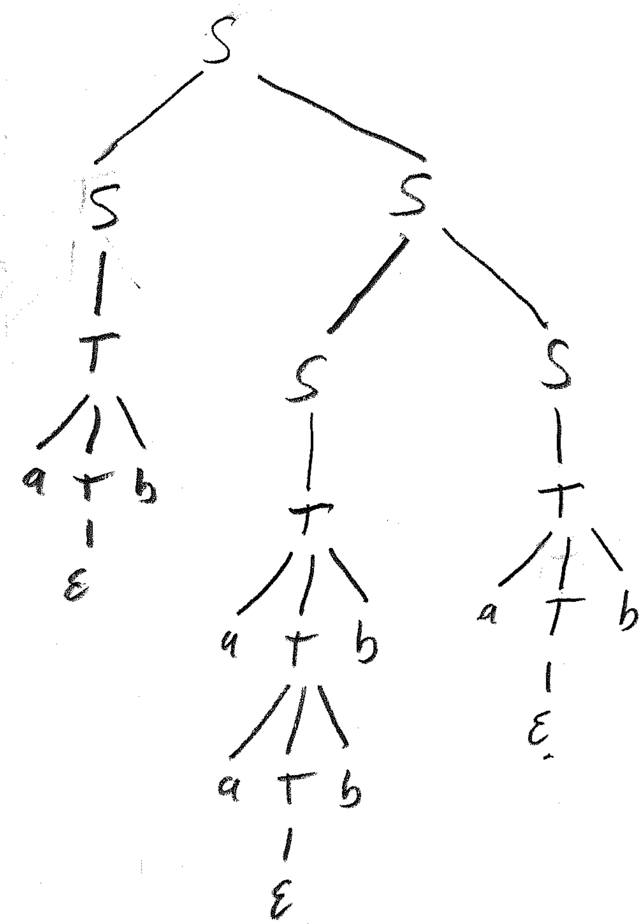
$\Rightarrow abaabb\underline{T}$

$\Rightarrow abaabba\underline{T}b$

$\Rightarrow abaabbab$

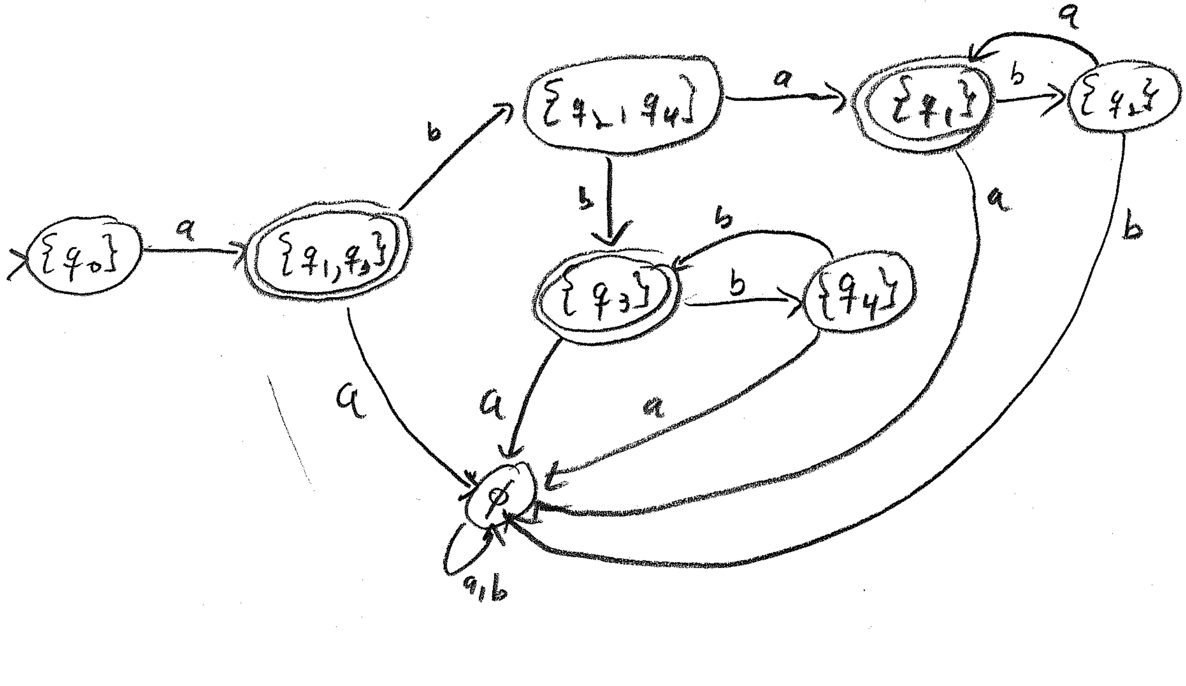


$$
\begin{aligned}
S &\Rightarrow \underline{S}\,S \\
&\Rightarrow \underline{T}\,S \\
&\Rightarrow a\underline{T}b\,S \\
&\Rightarrow ab\underline{S} \\
&\Rightarrow ab\underline{S}S \\
&\Rightarrow ab\underline{T}S \\
&\Rightarrow aba\underline{T}bS \\
&\Rightarrow abaa\underline{T}bbS \\
&\Rightarrow abaabb\underline{S} \\
&\Rightarrow abaabb\underline{T} \\
&\Rightarrow abaabba\underline{T}b \\
&\Rightarrow abaabbab
\end{aligned}
$$

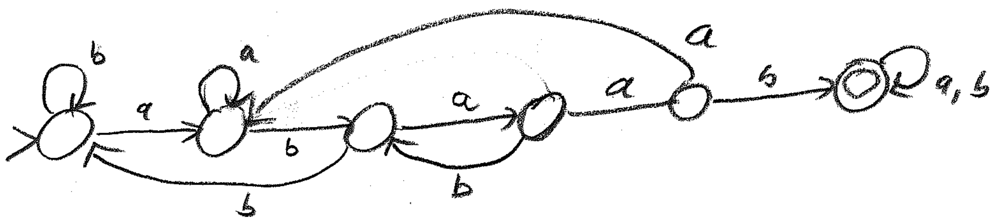


③ a) $a(ba)^* \mid a(bb)^*$ [or: $a((ba)^* \mid (bb)^*)$

b)



(4)



(5) $\{w \in \{a,b\}^* \mid w \text{ contains at least 2 a's}\}$: $b^*ab^*a(a|b)^*$

$\{w \in \{a,b\}^* \mid w \text{ contains exactly two a's}\}$: $b^*ab^*ab^*$

(6)

(7) Proof: To show $L \subseteq L^2$, we must show that for any w in L, w is also in L^2. Let $w \in L$. Since $w \in L$ and $\varepsilon \in L$, then by definition of L^2, $w\varepsilon \in L^2$. But $w\varepsilon = w$, so this shows $w \in L^2$. Therefore, $L \subseteq L^2$.

(8) $x^R = x$ means if you read the symbols in x in reverse order, you get the same thing as x (in the normal order). In other words, x is a palindrome, a string that reads the same forewards and backwards. For example: abccba, aabaa, abbbccbbba

(9)

$S \to aSa$
$S \to T$
$T \to bTb$
$T \to c$

The 1st rule can make strings of the form a^nSa^n, with the same number of a's before and after the S. The next rule changes this to a^nTa^n. The T can then be used to make equal numbers of b's to either side of the T, giving $a^n b^m T b^m a^n$. Finally, $T \to c$ changes the T into a c between the two groups of b's, giving $a^n b^m c b^m a^n$.

10. a) 5 b) 1 c) 3 d) 2 e) 4 f) 6

11. G defines a language consisting of strings over the alphabet Σ. The language contains strings of symbols from Σ that can be "derived" from the start symbol, S. To derive a string from S, we start from S and apply a sequence of one or more production rules, from the set P of production rules in G. A production rule has the form $A \to w$, where $A \in V$ and $w \in (V \cup \Sigma)^*$. To apply this production rule means to replace an A in a string with the string w. More formally, $L(G) = \{ x \in \Sigma^* \mid S \Rightarrow^* x \}$.

12. An NFA is nondeterministic because at some points in its computation, there might be more than one thing it can do. We can think of the NFA as choosing one of those possible moves at random. After an NFA has read part of a string, we can only ask what states it might possibly be in. An NFA accepts a string if it is <u>possible</u> for it to read the entire string and end in one of its accepting states.