Your work should be submitted through Canvas by 11:59 PM on Saturday, May 9.

About the exam: This exam counts for 30% of the total grade for the course. The work that you submit for this exam should be your own. You can use course materials, including the textbook, your notes, class notes and videos from online lectures, and posted solutions to homeworks. You should not use other books or material from the Internet. You can ask your professor questions about the exam, but you should not receive help on the exam from other students, your friends and family, or anyone else.

For the problems on this exam, you should not just find answers. You should present solutions. Write out your answers carefully, including explanations to justify your work when appropriate.

For the essay questions on this exam, you should write out clear and well-organized responses in complete sentences and paragraphs. The questions are meant to give you an opportunity to display your understanding of the central ideas from the course. Please type your answers to essay questions in a word processing application, if at all possible, and save your work as a PDF or as a Microsoft Word or Open Office document.

As usual, you can submit your work in Canvas in the form of PDFs, image files, and/or word processing documents.

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1. Basic Problems on Logic and Sets. (14 points.)

a) State the *negation* of the sentence, "If Albert says that the grass is pink, I believe him."

b) Express the following sentence in predicate logic, defining any predicates that you use: "Every black cat is unlucky, but not every unlucky cat is black."

c) Give a formal proof of the argument shown at the right. Give a reason for each step in the proof. If you are using a named result such as Modus Ponens, include the name.

$$\begin{array}{l} t \to p \\ q \to s \\ p \to (q \vee r) \\ t \\ \neg s \\ \hline \therefore r \end{array}$$

d) Write out the power set, $\mathcal{P}(A)$, if A is the set $A = \{a, b, \{a\}, \{a, b\}\}$.

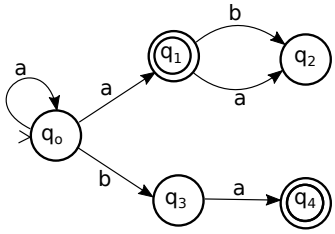
e) Suppose that L and M are languages over the alphabet $\{a, b, c\}$ defined as follows: $L = \{aa, ba, ca\}$ and $M = \{\varepsilon, b, c\}$. Find the language L^*M. (Give a specification of the strings in L^*M and explain your reasoning.)

2. Proofs. (12 points.)

a) Let A and B be sets. Give a proof by contradiction of the statement, if $A \times B = \varnothing$, then $A = \varnothing$ or $B = \varnothing$.

b) One of DeMorgan's laws for sets says that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Use a proof by induction to show that this law holds for any number of sets: $\overline{A_1 \cap A_2 \cap \cdots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \cdots \cup \overline{A_n}$. (The point here is to lay out a careful inductive argument.)

3. Regular expressions, DFAs and NFAs. (12 points.)



a) Find a regular expression for the language accepted by the NFA shown at the right.

b) Apply the NFA-to-DFA conversion algorithm to create a DFA that accepts the same language as the NFA shown at the right.

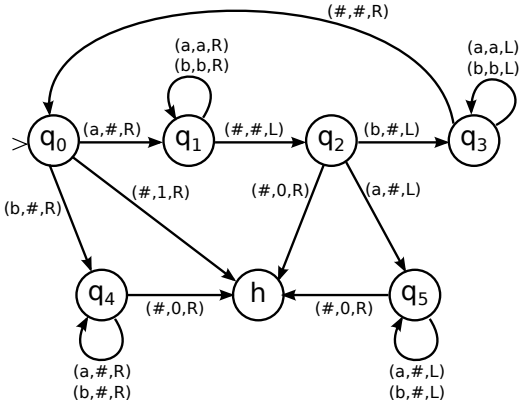
4. Grammars. (12 points.)

$$
\begin{aligned}
S &\longrightarrow XTbZ \\
T &\longrightarrow aTD \\
T &\longrightarrow \varepsilon \\
Db &\longrightarrow bbD \\
DZ &\longrightarrow Z \\
Xa &\longrightarrow aX \\
Xb &\longrightarrow bX \\
XZ &\longrightarrow \varepsilon
\end{aligned}
$$

a) What language is generated by the general grammar shown at the right? This grammar is very similar to one that you have seen before, in class or in the textbook. (Explain carefully, in detail, how the grammar works.)

b) Give a context-free grammar for the language $\{a^n b^m c^k \mid m > n+k\}$. Explain carefully how your grammar works and why it generates the given language.

5. Turing Machines. (10 points.) Assume that the Turing machine shown at the right is always started on the **left** end of a string of a's and b's.



a) If the machine is started on the string ab, what is on the tape when it halts? Why?

b) What if it is started on the string $abbbb$?

c) What if it is started on the string $aaabbb$?

d) In general, given any string of a's and b's, what does this machine output? Explain how the machine operates on an input string.

6. Essay Questions. (40 points.) Please type your answers in a word processing application, if possible, and submit either a PDF or a Microsoft Word or Open Office document. Remember to write your responses as well-organized essays in full sentences and paragraphs.

a) [7 points] One type of logic studied in this course was *predicate logic*. Explain what it meant by a predicate, and how predicates are used with quantifiers.

b) [13 points] This course covered several different types of language, organized into a language hierarchy. Explain what is meant by a "language" in this context, and discuss the discuss the various kinds of language that we have encountered and how they relate to each other.

c) [20 points] In this course, we studied several kinds of automata or abstract computing machines. One of the goals of this study was to help you understand what it means to compute and what can and cannot be accomplished by computation. Write an essay reviewing the various kinds of abstract machine that you have encountered in the course and what they can do, and explaining what you have learned from this study about the nature of computation.