This homework on Sections 4.6 and 5.1 is due on the last day of class, Monday, May 4.

1. Find a general grammar that generates the language $\{a^nba^nba^n \mid n \in \mathbb{N}\}$. Explain how your grammar works. Note: To explain how a grammar works, you can give a short explanation of the purpose of groups of rules or of individual rules. It can be useful to show typical strings produced at several stages of a derivation. Compare this problem to the grammar, shown at the right, for the language $\{a^nb^nc^n \mid n \in \mathbb{N}\}$, which was given as an example in class.

$$\begin{aligned}
S &\longrightarrow XTZ \\
T &\longrightarrow AbCT \\
T &\longrightarrow \varepsilon \\
bA &\longrightarrow Ab \\
CA &\longrightarrow AC \\
Cb &\longrightarrow bC \\
CZ &\longrightarrow Zc \\
XA &\longrightarrow aX \\
X &\longrightarrow \varepsilon \\
Z &\longrightarrow \varepsilon
\end{aligned}$$

2. Find a grammar for the language $\{a^{nm} \mid n, m \in \mathbb{N},\ n \geq 2,\ m \geq 2\}$. Explain how your grammar works. This grammar consists of strings of a's, where the number of a's is greater than 1 and is **not** a prime number. Compare this problem to the grammar, shown at the right, for $\{a^{n^2} \mid n \in \mathbb{N}\}$, which was given as an example in class.

$$\begin{aligned}
S &\longrightarrow XTZ \\
T &\longrightarrow BTC \\
T &\longrightarrow \varepsilon \\
BC &\longrightarrow CaB \\
Ba &\longrightarrow aB \\
aC &\longrightarrow Ca \\
BZ &\longrightarrow Z \\
XC &\longrightarrow X \\
X &\longrightarrow \varepsilon \\
Z &\longrightarrow \varepsilon
\end{aligned}$$

3. Using your grammar from the previous problem, give a derivation for the string $aaaaaa$. Note that this string is $a^{2\cdot 3}$.

4. Find a grammar for the language $\{a^nb^mc^{mn} \mid n, m \in \mathbb{N}\}$. Explain how your grammar works. (Note that the c^{mn} in this language is can be created in a similar way to the a^{mn} in problem **2**, but you also need to get a^n and b^m in their correct positions.)

For problems 5 and 6, you can either draw a transition diagram for a Turing machine or you can construct one in the Turing Machine web app that will be demonstrated in class on Wednesday, April 29. If you choose to use the web app, you should read the information and instructions at https://math.hws.edu/eck/cs229/s20/TM.html.

5. The textbook gives a transition diagram for a Turing machine that doubles the length of a string of a's. It does this by changing each a to a c, and adding an additional c to the string for each a it finds. At the end, it changes all of the c's back to a's. The machine is said to compute the function $f(a^n) = a^{2n}$. Create a Turing machine that computes the function $f(a^n) = a^{3n+1}$. That is, the machine is started at the left end of a string of a's. If there are n a's in the string, then when the machine halts, there should be $3n + 1$ a's on the tape.

6. Create a Turing machine that decides the language $\{w \in \{a,b\}^* \mid w = w^R\}$. This is the language of palindromes, which read the same backwards and forwards. That is, the machine is started on the left end of a string of a's and b's. If the string is a palindrome, then the machine should output a one; and if the string is not a palindrome, then the machine should output a zero. One way to approach this problem is to erase letters in pairs from the two ends of the string. If the character on the left end is an a, then the matching character on the right end must also be an a; if not, then the original string is not a palindrome, and the rest of the string should be erased and replaced by a 0. If the string is completely erased without finding any bad match, then the string was a palindrome, so the machine should write a 1 to the tape. You have to be careful to account for palindromes of odd length, which will have a single unmatched character in the middle.