This homework covers the reading for the second week of classes: Chapter 1, Sections 3, 4, and 5. It is due in class on Friday, February 7. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.

1. Draw the logic circuit that computes each of the following propositions. Do not simplify the circuit; build it directly from the Boolean expression.

a) $(A \wedge B) \vee ((\neg A) \wedge C)$

b) $(A \vee B \vee C) \wedge \neg(A \wedge B \wedge C)$

2. Express the following statements in predicate logic. Try to express as much of the meaning as you can. Give the meaning of each predicate that you use. When it is not clear, state the domain of discourse of your predicates. But remember that it's better to have a predicate that represents "is a cat" than to say "the domain of discourse is cats."

a) Every cat is selfish.

b) Every cat in Geneva is selfish.

c) There is a selfish cat in Geneva.

d) Everyone who owns a black cat is unlucky.

e) There is a book that no one has read.

3. Find the negation of each of the following expressions. Simplify the answer, so that the operator $\neg$ is only applied to individual predicates. (Show your work by writing a chain of logical equivalences, starting from the negation of the given statement.)

a) $\neg\big[\ \big(\forall x\,(P(x))\big) \vee \big(\forall x\,(Q(x))\big)\ \big]$

b) $\neg\,\forall x\,\big(P(x) \vee Q(x)\big)$

c) $\neg\,\forall n\,\big(\,Z(n) \rightarrow \exists k(Z(k) \wedge G(k,n))\,\big)$

4. Translate the following English sentence directly into predicate logic: "Not everyone who lives in Geneva is a student." Then simplify the resulting expression by applying the rules for negation. And finally, translate the result back into English.

5. Give a formal proof of that each of the following arguments is valid. State the justification for each step in the proof.

a)
$(p \wedge \neg q) \rightarrow r$
$q \rightarrow \neg p$
p
$\therefore\ r$

b)
$p \rightarrow r$
$(r \wedge s) \rightarrow t$
$q \rightarrow \neg t$
s
q
$\therefore\ \neg p$

c)
$(r \wedge s) \rightarrow p$
$\neg(p \wedge q)$
r
q
$\therefore\ \neg s$

6. Translate each of the following arguments, expressed in English, into logic, and determine whether the argument is valid. If the argument is valid, give a formal proof. If it is not valid, explain why.

a) If this card is red, then it's the King of Hearts. But if this card is a King, then it is not a Heart. So, this card is not red.

b) A math major takes Abstract Algebra or Foundations of Analysis. If someone takes Abstract Algebra, then that person knows about Galois. Mary graduated with a major in math, but she never took Foundations of Analysis. So, Mary knows about Galois.

c) If Bill stays up late partying, he is tired the next day. If Bill is tired and there is a test, he doesn't do well on the test. If Bill does well on a test, he celebrates. There was a test today, and Bill is not celebrating, so he must have stayed up late partying last night.