This homework covers Chapter 1, Sections 6, 7, and 8. It is due in class on Friday, February 14.

For the proofs that you are asked to give on this homework, you should give informal, but careful and complete, proofs of the kind that are typically given by mathematicians. In your proofs, you can use the following facts without proving them:

- *The product of any two rational numbers is rational.*
- *The sum of any two rational numbers is rational.*
- *The number $\sqrt{2}$ is irrational.*

You can also use basic facts from algebra.

1. Suppose that a, b, and c are integers such that $a \mid b$. Prove that $a \mid (bc)$.

2. Prove or disprove:

a) For any real number x, if x^2 is an irrational number, then x is also irrational.

b) For any real number x, if x is an irrational number, then x^2 is also irrational.

3. Prove or disprove:

a) If n is an integer and n is divisible by 4, then n^2 is divisible by 4.

b) If n is an integer and n^2 is divisible by 4, then n is divisible by 4.

4. Prove that for any integer n, the number $n^2 + n$ is even. (Consider a proof by cases, looking at the case where n is even and the case where n is odd.)

5. Write out each of the following sums in full, without using summation notation. You are **not** being asked to compute the value of the sums!

a) $\displaystyle\sum_{j=0}^{5} (2j+1)$ **b)** $\displaystyle\sum_{n=1}^{6} \frac{1}{2^n}$ **c)** $\displaystyle\sum_{i=5}^{8} \frac{i-1}{i+1}$

6. Use a proof by induction to show that for any integer $n \geq 1$, $\displaystyle\sum_{i=1}^{n} (2i-1) = n^2$

7. Use a proof by induction to show that for any integer $n \geq 0$, $\displaystyle\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$