

*This homework is due in class on Monday, February 24.*

1. Let  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ;  $B = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ ;  $C = \{n \in \mathbb{Z} \mid -5 \leq n \leq 5\}$ . Find the following sets. (For this exercise, you do **not** need to justify your answers.)

- a)  $A \cup B$       b)  $A \cap B$       c)  $A \setminus B$       d)  $B \setminus A$   
 e)  $A \cap C$       f)  $\mathbb{N} \cup C$       g)  $\mathbb{N} \setminus C$       h)  $\mathbb{Z} \setminus A$

(Recall that  $\mathbb{N}$  is the set of natural numbers and  $\mathbb{Z}$  is the set of integers.)

2. (Exercise 2.1.7 from the textbook.) In the English sentence, “She likes men who are tall, dark, and handsome,” does she like an intersection or a union of sets of men? How about in the sentence, “She likes men who are tall, men who are dark, and men who are handsome”? **Explain!**

3. Prove using the definitions of  $\cap$  and  $\subseteq$ : For any sets  $A$  and  $B$ , if  $A \cap B = B$ , then  $B \subseteq A$

4. Prove or disprove: For any sets  $A$  and  $B$ ,  $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

5. Let  $k$ ,  $n$ , and  $m$  be values of type *int* given as hexadecimal numbers in Java as

`k = 0xFF00`

`m = 0xAB24`

`n = 0x77`

Find the values of the following Java expressions, writing the answers as hexadecimal numbers.

- a) `k >> 4`      b) `(k & m) | n`      c) `(m << 16) | k | n`      d) `m & n`

6. Consider the two 16-bit integers  $n$  and  $m$  shown below. Compute the three 16-bit integers  $\sim n$ , and  $n \& m$ , and  $n | m$ . Also, write out the subset of  $\{0, 1, 2, \dots, 15\}$  that is represented by each of the integers  $n$ ,  $m$ ,  $\sim n$ ,  $n \& m$ , and  $n | m$ . (Write out each set in set notation.)

$n = 1001\ 1101\ 1000\ 0101$

$m = 0101\ 1001\ 1100\ 0111$

7. What is computed by the following method? (Hint: Write  $N$  in binary.) **Explain your answer!**

```
int countSomething( int N ) {
    int ct = 0;
    for (int i = 0; i <= 31; i++) {
        if ( (N & 1) == 1 ) {
            ct++;
        }
        N = N >> 1;
    }
    return ct;
}
```

8. Consider the following functions from  $\mathbb{N}$  to  $\mathbb{N}$ . (Recall that  $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$  — and that 0 is an even number.) For each function, determine whether the function is *one-to-one* and whether the function is *onto*. **Justify your answers!**

a)  $f(n) = 2n$

b)  $g(n) = n/2$  (using the Java definition that  $n/2$  is the integer quotient)

c)  $h(x) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$