This homework on Sections 2.6, 3.1, 3.2, and 3.3 is due in class on Friday, March 13.

1. Suppose that A, B, and C are sets and that their cardinalities are $|A| = 7$, $|B| = 3$, and $|C| = 5$. Find the following cardinalities:

a) $|A \times B \times C|$ **b)** $|\mathcal{P}(B \times C)|$ **c)** $\left|\mathcal{P}(A) \times \mathcal{P}(B)\right|$

d) What is the range of possible values for $A \cup B \cup C$? Why?

2. Use proof by contradiction to prove each of the following statements:

a) Suppose that X is a countably infinite set and N is a finite subset of X. Then $X \setminus N$ is infinite.

b) Suppose that X is an infinite set and A is a subset of X. Then at least one of A and $X \setminus A$ is infinite.

3. Suppose that languages M and L over the alphabet $\{a, b\}$ are defined as

$$M = \{\varepsilon, a, ab\} \qquad L = \{w \in \{a, b\}^* \mid w \text{ ends with a } b\}$$

Find each of the following languages. Write your answers using either set notation or a clear English specification of the language. If the answer is not obvious, then you should justify your answer in words.

a) M^2 **b)** M^* **c)** LM **d)** ML **e)** L^R **f)** L^2 **g)** L^*

4. Give an English description of the language generated by each of the following regular expressions over the alphabet $\{a, b\}$, or write out the answer as a set:

a) bab^* **b)** $b(ab)^*$ **c)** $(bab)^*$

d) $(a|b)^*bbb(a|b)^*$ **e)** $a^*ba^*ba^*ba^*$ **f)** $a^*(b|\varepsilon)a^*(b|\varepsilon)a^*(b|\varepsilon)a^*$

5. Write a regular expression for each of the following languages. If the answer is not obvious, justify your answer by explaining how your regular expression works.

a) $L_1 = \{w \in \{a, b\}^* \mid w \text{ begins with } ab \text{ and ends with } ba\}$

b) $L_2 = \{w \in \{a, b\}^* \mid |w| \geq 2 \text{ and } w \text{ begins and ends with the same symbol}\}$

c) $L_3 = \{w \in \{a, b, c\}^* \mid w \text{ contains a } b \text{ and there are no } c\text{'s before the first } b \text{ in } w\}$

d) $L_4 = \{w \in \{a, b, c\}^* \mid \text{the number of } a\text{'s in } w \text{ is a multiple of 3}\}$

e) $L_5 = \{w \in \{a, b, c\}^* \mid \text{the number of } a\text{'s in } w \text{ is } \textbf{not} \text{ a multiple of 3}\}$

6. This problem uses the extended regular expression syntax from Section 3.3, and the exercises come from that section. You can also use patterns from the regular expression handout.

a) Write a regular expression that will match a ten-digit phone number written in the form (xxx)xxx-xxxx, where each "x" is one of the digits 0 through 9, except that the very first x cannot be 0 or 1.

b) Write a search pattern and a corresponding replace pattern that can be used to replace a ten-digit phone number in the form (xxx)xxx-xxxx with the name number written using the syntax xxx-xxx-xxxx. (For the search pattern, you can use your answer from part *a*, with parentheses added for grouping.)

c) Write a regular expression that will match a Java comment consisting of `//` and everything following it on the same line.

d) Write a search pattern and a corresponding replace pattern that can be used to replace a comment consisting of the characters between `//` and end of line with a comment consisting of the same characters between `/*` and `*/`.