CPSC 229, Spring 2020 **Homework #8**

This homework on Sections 3.7 and 4.1 is due on Wednesday, April 15.

1. For each of the following languages, use the Pumping Lemma to prove that the language is not regular.

a) The language $\{a^n b^m c^n \mid n, m \in \mathbb{N}\}$. Note that the number of a's equals the number of c's.

b) The language of palindromes over the alphabet $\{a, b\}$: $\{w \in \{a, b\}^* \mid w = w^R\}$.

c) The language $\{a^n b^m c^k \mid k = mn\}$.

(Here is the template that I showed in class for doing a Pumping Lemma proof: *Proof that a language, L, is not regular:* Assume, for the sake of contradiction, that L is regular. Then by the Pumping Lemma, there is an integer K such that for any $w \in L$ with $|w| \geq K$, w can be written $w = xyx$ where $|xy| \leq K$, $y \neq \varepsilon$, and $xy^n z \in L$ for all $n \in \mathbb{N}$. But let $w =$ ______, which is in L with length greater than or equal to K. Write $w = xyz$ as in the Pumping Lemma. Because $|xy| \leq K$ (meaning xy is part of the first K characters in w), we see that y must be of the form ______. But then $xy^n z \notin L$ for $n =$ ______, because ________________________.)

2. Consider the context free grammar shown at the right.

$$\begin{aligned} S &\longrightarrow aaTb \\ T &\longrightarrow aT \\ T &\longrightarrow aTb \\ T &\longrightarrow \varepsilon \end{aligned}$$

a) Write a derivation for the string $aaaab$ using this grammar.

b) Write a derivation for the string $aaaaabbb$ using this grammar.

c) Find the language generaged by this grammar. Briefly justify your answer.

3. Consider the context free grammar shown at the right.

$$\begin{aligned} S &\longrightarrow TR \\ T &\longrightarrow aTb \\ T &\longrightarrow \varepsilon \\ R &\longrightarrow cRd \\ R &\longrightarrow c \end{aligned}$$

a) Write a derivation for the string $aabbc$ using this grammar.

b) Write a derivation for the string $abcccdd$ using this grammar.

c) Find the language generated by this grammar. Briefly justify your answer.

4. For each of the following languages, create a Context-Free Grammar that generates that language. Explain in words why your grammar works; you should explain in enough detail that it is clear that your grammar can generate all of the strings in the language and only the strings in the language. For Part (b), a hint is that the language can be seen as the union of two simpler languages. Part (d) is particularly tricky. Think of the non-terminal symbols as being something like states, and think about how you need to change from one state to another to make sure that you are generating the right symbols in the correct order and the correct number.

a) $\{a^n b a^m \mid m \geq n > 0\}$

b) $\{a^n b^m \mid n \neq m\}$

c) $\{a^n b^m c^k d^l \mid m = k \text{ and } n = l\}$

d) $\{a^n b^m c^k d^l \mid n + m = k + l\}$