*The first test for this course will be given in class on Wednesday, February 26. It covers all of the material that we have done in Chapters 1 and 2, through Section 2.4.*

*The test will include some "short essay" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework. You can expect to do a few simple proofs, including at least one formal proof of the validity of an argument and at least one more informal mathematical proof. There might be a simple proof by induction.*

## Here are some terms and ideas that you should be familiar with for the test:

translations from logic to English, and from English to logic

propositions; propositional logic

the logical operators "and" ($\wedge$), "or" ($\vee$), and "not" ($\neg$)

some information is lost when translating English to logic (for example, "but" vs. "and")

truth table

logical equivalence ($\equiv$)

the conditional or "implies" operator ($\rightarrow$)

definition of $p \rightarrow q$ as $(\neg p) \vee q$

negation of an implication: $\neg(p \rightarrow q) \equiv p \wedge \neg q$

the biconditional or "if-and-only-if" operator ($\leftrightarrow$)

tautology

Boolean algebra

some basic laws of Boolean algebra (double negation, De Morgan's, commutative, etc.)

converse of an implication ($p \rightarrow q$ has converse $q \rightarrow p$)

contrapositive of an implication ($p \rightarrow q$ has contrapositive $(\neg q) \rightarrow (\neg p)$)

an implication is logically equivalent to its contrapositive

predicates; predicate logic

one-place predicate, two-place predicate, etc.

domain of discourse

quantifiers, "for all" ($\forall$) and "there exists" ($\exists$)

negation of a statement that uses quantifiers, such as $\neg\exists x\,(P(x)) \equiv \forall x\,(\neg P(x))$

arguments and logical deduction

premises and conclusion of an argument

what it means for an argument to be valid

formal proof of the validity of an argument

how to show that an argument is invalid

translating arguments from English into logic

Modus Ponens, Modus Tollens, and Elimination

mathematical proof

proving a universally quantified statement, $\forall x, P(x)$

direct proof of a statement using $\rightarrow$, such as $\forall x(P(x) \rightarrow Q(x))$,

existence proof
counterexample
proof by contradiction
even and odd numbers
rational number (real number that can be expressed as a quotient of integers, $\frac{a}{b}$)
irrational number (real number that is not rational such as $\pi$ or $\sqrt{2}$)
divisibility (for integers $n$ and $m$, $n$ is divisible by $m$ if $n = km$ for some integer $k$)
the notation $a \mid b$ meaning $b$ is divisible by $a$
prime number (greater than 1, and cannot be factored into smaller positive integers)
proof by mathematical induction (first form only)
summation notation, for example: $\sum_{k=1}^{n} a_k$
sets
set notations: $\{a, b, c\}$, $\{1, 2, 3, \dots\}$, $\{x \mid P(x)\}$, $\{x \in A \mid P(x)\}$
the empty set, $\varnothing$ or $\{\,\}$
equality of sets: $A = B$ if and only if they contain the same elements, $\forall x(x \in A \leftrightarrow x \in B)$
element of a set: $a \in A$
subset, $A \subseteq B$: $\forall x(x \in A \rightarrow x \in B)$
for sets, $A = B$ is equivalent to $A \subseteq B$ and $B \subseteq A$
union, intersection, and set difference: $A \cup B$, $A \cap B$, $A \smallsetminus B$
definition of set operations in terms of logical operators
power set of a set: $\mathcal{P}(A)$
universal set
complement of a set (in a universal set): $\overline{A}$
DeMorgan's Laws for sets: $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$
bitwise operations in Java: `&, |, ~`
correspondence between $n$-bit binary numbers and subsets of $\{0, 1, 2, \dots, n-1\}$
`&`, `|`, and `~` as set operations (intersection, union, complement)
the shift operators `<<`, `>>`, and `>>>`
hexadecimal numbers
ordered pair: $(a, b)$
cross product of sets: $A \times B$
function $f\colon A \rightarrow B$
one-to-one function (also called injective function)
onto function (also called surjective function)
one-to-one correspondence (also called bijective function)

**Here are some problems from tests given in previous courses:**

*These questions are just examples to give you an idea of some types of questions that might be on the test. This is not meant as a completely comprehensive review, and you should not expect every question on the test to be similar to one of these. Note that there are more questions here than would fit onto a single test.*

**1.** Define the following terms, as they relate to this course:

**a)** *logical equivalence* of propositions

**b)** *subset*

**c)** the *cross product*, $A \times B$, of two sets

**2.** Let $p$ be the following proposition: "If global temperatures rise by two degrees, then the Greenland ice sheet melts."

**a)** State the *converse* of $p$ (in natural English)

**b)** State the *contrapositive* of $p$ (in natural English)

**c)** State the *negation* of $p$ (in natural English):

**3.** Use a truth table to show that the proposition $(p \wedge q) \rightarrow r$ is logically equivalent to the proposition $p \rightarrow (q \rightarrow r)$. (Don't forget to say what it is about the truth table that demonstrates the equivalence.)

**4.** Consider the following propositions, where the domain of discourse in all cases is the set of people:

$R(x)$ stands for "$x$ is rich"
$H(x)$ stands for "$x$ is happy"
$L(u, v)$ stands for "$u$ likes $v$"

**a)** Translate the sentence "Not everyone is happy" into predicate logic.

**b)** Translate the sentence "All rich people are happy" into logic.

**c)** Translate the sentence "There is an unhappy rich person" into logic.

**d)** Translate $\forall x\, \big(R(x) \rightarrow \forall y\, L(y, x)\big)$ into an unambiguous sentence in English.

**e)** Translate $\forall x\, \big((\exists y\, L(y, x)) \rightarrow H(x)\big)$ into an unambiguous sentence in English. ( This one really should have been $\forall x\, \big((\exists y\, (y \neq x \wedge L(y, x))) \rightarrow H(x)\big)$ )

**5.** Simplify the following, so that in the end the $\neg$ operator is applied only to individual predicates.

$\neg(\exists x\, \big(P(x) \wedge \forall y\, (Q(x, y) \vee R(x, y)\big)$

**6.** Give a *formal proof* that the following argument is valid. Don't forget to give a reason for each step in the proof.

$(q \wedge (\neg s)) \rightarrow p$
$s \rightarrow t$
$\neg t$
$q$

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$\therefore p$

**7.** Consider the proposition $\forall x \, \exists y \, L(x, y)$ and the proposition $\exists y \, \forall x \, L(x, y)$. Carefully explain the difference in the meanings of these two propositions. It will be helpful to use some specific predicate $L$ as an example.

**8.** Give a *proof by contradiction* of the following statement: If $x^2$ is an odd integer, then $x$ is also odd.

**9.** Prove that for any integers $a$, $b$, and $c$, if $a$ is divisible by $c$ and $b$ is divisible by $c$, then the sum $a + b$ is also divisible by $c$.

**10.** Let $A$, $B$, and $C$ be the sets:

$$A = \{2, 3, 5, 7, 11, 13\} \qquad B = \{1, 2, 3\} \qquad C = \{a, b\}$$

Find the following sets:

**a)** $A \cap B =$ ____________________________

**b)** $A \smallsetminus B =$ ____________________________

**c)** $C \times B =$ ____________________________

**d)** $\mathcal{P}(C) =$ ____________________________

**11.** Let $X$ be the set $X = \big\{ a, \{b\}, \{a, b\}, \{a, \{b\}\}, \{a, \{a\}\} \big\}$.

**a)** Is the set $\{a\}$ an *element* of $X$ or a *subset* of $X$ or neither or both? Why?

**b)** Is the set $\{a, b\}$ an *element* of $X$ or a *subset* of $X$ or neither or both? Why?

**c)** Is the set $\{a, \{b\}\}$ an *element* of $X$ or a *subset* of $X$ or neither or both? Why?

**12.** Suppose that $n$ and $m$ are variables of type *int* in a Java program, with values $n =$ `0xABCDEF` and $m =$ `0x123456`. What is the value of the following expression? (Explain your answer!)

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(n & 0xFFF000) | (m & 0xFFF)
```

**13.** What is meant by a *universal set*? Where are universal sets used, and why are they necessary? Give an example that uses a universal set.

**14.** Discuss the principle of mathematical induction. (What does it say? Why is it true, intuitively? What is meant by the base case and the inductive case of an induction?)