This is the homework for the week of September 8–12, covering Chapter 1, Sections 4 and 5. It is due in class on Wednesday, September 17. You can work with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will not receive any credit.

1. Write the *negation* of each of the following statements in simplified form. In your answer, the *not* operator, $\neg$, should only be applied to simple terms, such as $\neg P(a)$ or $\neg L(x, y)$.

a) $\exists x(\neg P(x))$ **b)** $\exists x(P(x) \wedge Q(x))$

c) $\forall x \, \forall y \, L(x, y)$ **d)** $\forall y(W(y) \rightarrow H(y))$

e) $\forall x \, \exists y \forall z \, (P(x, y) \vee Q(y, z))$ **f)** $\forall x\big(Q(x) \rightarrow (\exists y \, T(x, y))\big)$

2. The sentence "Someone has the answer to every question" is ambiguous. Give two translations of this statement into predicate logic, and carefully explain the difference in meaning.

3. Translate each of the following English sentences into predicate logic. Make up any predicates that you need. State what each predicate means and what its domain of discourse is.

a) Not everyone likes Keanu Reeves.

b) All upper level math classes are fun.

c) Some lower level math class is boring.

d) The United States has exactly one president.

4. Give a formal proof of each that each of the following arguments is valid. State the justification for each step in the proof.

a)
$p \rightarrow r$
$(r \wedge s) \rightarrow w$
$(\neg s) \rightarrow q$
$\neg q$
p
---
w

b)
$r \rightarrow s$
$s \rightarrow (t \wedge p)$
$\neg p$

$\neg r$

c)
$(p \wedge q) \rightarrow (r \vee s)$
$\neg r$
$p \rightarrow q$
p
---
s

5. Translate each of the following arguments, expressed in English, into logic, and determine whether the argument is valid.

a) If it's raining then Jim doesn't play golf. If Jim doesn't play golf, he's not happy. Jim's happy. So It's not raining.

b) If Jane is sick, then she stays home from work. Jane is not at work today, so she must be sick.

c) In order to get a B.S. degree, you must pass either a math class or a computer science class. If you don't understand algebra, you can't pass a math class. Mary has a B.S. degree, but Mary doesn't understand algebra. So Mary must have taken a computer science class.