This is the homework for the week of September 15–19, covering Chapter 1, Sections 7 and 9. It is due in class on Wednesday, September 24. You can work with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will not receive any credit.

Please remember that there will be a test on Monday, September 29.

1. Translate each of the following mathematical statements into predicate logic, keeping in mind that in mathematics, "for all" is often assumed. Then prove or disprove each statements. (Taken from exercise 7, page 48.)

a) The product of two even integers is even.

b) If the product of two even integers is even, then both of the integers are even.

c) The product of two rational numbers is rational.

d) The product of two irrational numbers is irrational.

e) For all integers n, if n is divisible by 4, then n^2 is divisible by 4.

f) For all integers n, if n^2 is divisible by 4, then n is divisible by 4.

2. Write the following sums without using summation notation:

a) $\displaystyle\sum_{i=0}^{4} 3^i$ **b)** $\displaystyle\sum_{k=3}^{8} \frac{k^2+1}{k}$

3. The distributive law for arithmetic says that for any numbers x, y, and z, we have that $z(x+y) = zx + zy$. Give a proof by mathematical induction to show that for any numbers, and for any positive integer n,

$$z(x_1 + x_2 + \cdots + x_n) = zx_1 + zx_2 \cdots + zx_n$$

(Use induction on n.)

4. Give a proof by mathematical induction to show that for any positive integer n,

$$\sum_{k=1}^{n} 3^k = \tfrac{3}{2}\left(3^n - 1\right)$$

5. Give a proof by mathematical induction to show that for any positive integer n,

$$\sum_{i=1}^{n} i \cdot 2^{i-1} = (n-1)2^n + 1$$