

This homework covers Chapter 2, Sections 4 and 5. It is due in class on Friday, October 17. You can work with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will not receive any credit.

Note that there is no class on Monday, October 13.

1. Let A and B be sets, and suppose that $A \times B = B \times A$. Does this imply that $A = B$? Explain.
2. Consider the following functions. For each function, decide whether the function is one-to-one and whether it is onto. *Justify your answers.* (Remember that the answers depend on the domain and range of the function, not just on the formula that defines it.)
 - a) $f: \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(n) = n + 1$
 - b) $g: \mathbb{N} \rightarrow \mathbb{N}$, defined by $g(n) = \begin{cases} n - 1 & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$
 - c) $h: \mathbb{Z} \rightarrow \mathbb{Z}$, defined by $h(x) = 2x$
 - d) $s: \mathbb{R} \rightarrow \mathbb{R}$, defined by $s(x) = 2x$
3. (Exercise 7 from Section 2.7.) Suppose that $f: A \rightarrow B$ and that $g: B \rightarrow C$, and suppose that the composition, $g \circ f$, is an *onto* function.
 - a) *Prove* that g is an onto function.
 - b) Find a specific counterexample to show that f is not necessarily onto.
4. (Exercise 3 from Section 2.8.) It is possible to define new types in Java. For example, the definition

```
class Point {  
    int x;  
    int y;  
}
```

defines a new type named *Point*. A value of type *Point* contains two values of type *int*. What mathematical operation corresponds to the construction of this data type? Why?