This homework is on Sections 2.6, 3.1, and 3.2. It is due in class on Monday, October 27. I will have a solution sheet for you at that time. Remember that there is a test on Wednesday, October 29. There will be office hours on Sunday, October 26, from 12:00 to 3:00.

You can work with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will not receive any credit.

1. Let A, B and C be sets and suppose that $|A| = 3$, $|B| = 7$ and $|C| = 5$. Compute the cardinality of each of the following sets (using Theorem 2.8):

a) $C \times C$ **b)** $A \times B \times C$ **c)** $\mathcal{P}(B) \times \mathcal{P}(A)$ **d)** $\mathcal{P}(B \times A)$

2. What is the range of possible values of $|B \cup C|$ where B and C are the same as in the previous problem? Why?

3. Consider the following languages over the alphabet $\{a, b, c\}$:

$$L = \{\varepsilon, a, b, c\}, \quad M = \{aa, bb, cc\}, \quad S = \{c, cb, cbb, cbbb, cbbbb, \dots\}$$

Find the following languages:

a) L^2 **b)** LM **c)** M^* **d)** S^R **e)** S^2 **f)** S^*

4. Give an English description of the language generated by each of the following regular expressions over the alphabet $\{a, b\}$:

a) ab^* **b)** $(ab)^*$ **c)** $(a + b)^*bbb(a + b)^*$ **d)** $a^*ba^*ba^*ba^*$

5. Find a regular expression that generates each of the following languages over the alphabet $\{0, 1\}$:

a) $\{w \in \Sigma^* \mid w \text{ ends with } 0101\}$
b) $\{w \in \Sigma^* \mid w \text{ starts and ends with the same symbol}\}$
c) $\{w \in \Sigma^* \mid |w| \geq 3\}$

6. In this problem, you will find a bijection between the set of real numbers between 0 and 1 and a certain subset of $\mathcal{P}(\mathbb{N})$.

a) The 16-bit binary number 1001111010100111 can be thought of as representing a certain subset of the set $\{0, 1, 2, \dots, 15\}$. What specific subset does it represent? Why? (See Section 2.3.)

b) An infinite sequence of bits, such as 10101010101010101..., can be thought of as representing a subset of $\mathbb{N}$. How? What particular subset does the infinite binary sequence 10101010101010101... represent?

c) Let X be the set $X = \{x \in \mathbb{R} \mid 0 < x \leq 1\}$. Every element of X. can be represented as an infinite binary decimal number such as 0.1010101010.... (The number 1 is represented as 0.111111111111....) Furthermore, we can assume that the binary decimal representation does not end in an infinite number of 0's (since any such number has another representation that ends in an infinite number of 1's). Explain why this representation defines a function $f\colon X \to \mathcal{P}(\mathbb{N})$ whose image contains all the *infinite* subsets of $\mathbb{N}$.

d) Explain why the function in part **c)** is a bijection.