This is the homework for the week of August 27–31, covering Chapter 1, Sections 1 to 3. It is due in class on Wednesday, September 5. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.

1. Construct a truth table for each of the following compound propositions, and use it to determine whether the proposition is a tautology:

a) $((p \to q) \wedge \neg p) \to \neg q$ **b)** $\big((p \vee q) \wedge \neg(q \vee r)\big) \to p)$

2. Consider an ordinary deck of 52 playing cards. A *face card* is a card that is either a Jack, a Queen, or a King. For how many cards in the deck is it true

a) that "This card is both a face card and a spade"?

b) that "This card is either a face card or a spade"?

c) that "This card is a face card if and only if it is a spade"?

d) that "If this card is a spade, then it is a face card"?

3. In Java, it is possible to compare two *boolean* values using the equality operator, `==`, and the inequality operator, `!=`.

a) Which of the logical operators that we have studied ($\wedge$, $\vee$, $\neg$, $\to$, and $\leftrightarrow$) has the same meaning as the Java operator `==` (applied to boolean values)? Explain.

b) Suppose that `P` and `Q` are *boolean* variables in Java. Write an expression that is equivalent to `P != Q` using only Java's basic logical operators `&&`, `||`, and `!`.

4. Draw a logic circuit that computes the value of the following propositions from its three inputs, A, B, and C. Describe in words how you constructed the circuit

$$(A \vee B \vee C) \wedge \neg(A \wedge B) \wedge \neg(A \wedge C)$$

5. Convert each of the following English statements into propositional logic. You should introduce symbols such as p and q to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for. The first statement is *ambiguous*; you should give two possible translations and explain the difference.

a) My next car will be red or black and white.

b) If I have to listen to another one of Fred's jokes, then I will either kill myself or kill him.

6. Express the logical negation of each of the following sentences in natural English

a) I don't eat oysters.

b) He likes Tobey McGuire, but he doesn't like Spiderman.

c) If global warming gets worse, then the Greenland ice sheet will melt.

7. State the *converse* and the *contrapositive* of the English statement, "If it's January, then it's snowing in Geneva." Explain in your own words why this statement and its converse are not logically equivalent.

8. Let's say that a logical operator $\uparrow$ is defined as follows: For any propositions A and B, the expression $A \uparrow B$ has the same logical value as the expression $\neg(A \wedge B)$. (In computer science, this operator is usually called `NAND`, which is short for "not and.") For each of the three logical expressions $\neg p$, $p \wedge q$, and $p \vee q$, show how to write an equivalent expression that uses no logical operator other than $\uparrow$. (You might need some hints: The expression that is equivalent to $\neg p$ contains p **twice**. Once you know how to write $\neg p$ in terms of $\uparrow$, you can use that to help you with $p \wedge q$ and $p \vee q$.)