CPSC 229, Fall 2007 — Homework #2

This homework is due in class on Wednesday, September 12. Remember that you can work with other people in the class, but you should write up your own solutions to turn in. Don't forget to show your work and explain you reasoning, if you want to get full credit for your answers.

1. Find the negation of each of the following propositions. In your answer, the negation operator should be applied only to simple terms such as $\neg P(a)$ or $\neg L(x,y)$

a) $\exists x\,(\neg R(x))$
b) $\exists z\,\big(S(z) \vee \neg T(z)\big)$
c) $\forall x\,\exists y\big(P(x) \vee Q(x,y)\big)$
d) $\forall x\,\big(C(x) \rightarrow B(x)\big)$
e) $\exists s\,\big(P(x) \wedge \forall t(Q(s,t)\big)$
f) $\forall x\,\forall y\,\big(G(y,x) \rightarrow \exists z\,(G(z,x) \wedge G(y,z)\big)$

2. Consider proposition **f** from the previous problem: $\forall x\,\forall y\,\big(G(y,x) \rightarrow \exists z\,(G(z,x) \wedge G(y,z)\big)$. Suppose that $G(a,b)$ means "a is greater than b" and that the domain of discourse is real numbers. With this interpretation, what does proposition **f** say about the real numbers? Is this proposition a true statement about the real numbers? Suppose that the domain of discourse for the proposition is the integers; is the proposition a true statement about the integers? Explain your answers.

3. The statement "Someone has the answer to every question" is ambiguous. Give *two* translations of this statement into predicate logic, and explain the difference in meaning. Then find the negation of each of your translations, and state the meaning of each negation in natural, unambiguous English.

4. Give a formal proof that each of the following arguments is valid. State the justification for each step in the proof.

a)
$$\begin{array}{l} p \rightarrow r \\ (r \wedge s) \rightarrow w \\ (\neg s) \rightarrow q \\ \neg q \\ p \\ \hline \therefore\ w \end{array}$$

b)
$$\begin{array}{l} r \rightarrow s \\ s \rightarrow (t \wedge p) \\ \neg p \\ \hline \therefore\ \neg r \end{array}$$

c)
$$\begin{array}{l} (p \wedge q) \rightarrow (r \vee s) \\ \neg r \\ p \rightarrow q \\ p \\ \hline \therefore\ s \end{array}$$

5. Translate each of the following arguments, expressed in English, into logic, and determine whether the argument is valid.

a) If it's raining then Jim doesn't play golf. If Jim doesn't play golf, he's not happy. Jim's happy. So It's not raining.

b) If Jane is sick, then she stays home from work. Jane is not at work today, so she must be sick.

c) In order to get a B.S. degree, you must pass either a math class or a computer science class. If you don't understand algebra, you can't pass a math class. Mary has a B.S. degree, but Mary doesn't understand algebra. So Mary must have taken a computer science class.