CPSC 229, Fall 2007 — Homework #3

Reminder: There is a test on Friday, September 21. The test will cover Chapter 1, Sections 1 through 7.

For the proofs that you are asked to do in this homework, you can give informal (but careful) proofs of the type that are typically given by mathematicians. You are not *required to give formal proofs.*

This homework is due in class on Wednesday, September 19. Remember that you can work with other people in the class, but you should write up your own solutions to turn in. Don't forget to show your work and explain you reasoning, if you want to get full credit for your answers.

1. Prove that there is no largest integer.

2. Suppose that a, b, and c are real numbers such that $a + b + c$ is greater than 30. Use a proof by contradiction to show that at least one of the numbers a, b, or c is greater than 10.

3. Prove that for any integer n, n is evenly divisible by 3 if and only if n^2 is evenly divisible by 3. (You can use the fact that every integer can be written as $3a$ or as $3a + 1$ or as $3a + 2$, for some integer a.)

4. Prove or disprove:

a) For any integer n, if n is divisible by 4, then so is n^2.

b) For any integer n, if n^2 is evenly divisible by 4, then so is n.

5. Show that the product, rs, of any two rational numbers r and s is also a rational number. (See the proof in the book that the sum of any two rational numbers is rational.)

6. Prove or disprove: The sum of any two irrational numbers is also an irrational number.