This homework is due in class on Wednesday, October 3. Don't forget to show your work and explain your reasoning, if you want to get full credit for your answers.

1. Use mathematical induction to prove that for any integer $n \geq 1$,

$$3^1 + 3^2 + \cdots + 3^n = \tfrac{3}{2}\big(3^n - 1\big)$$

2. The distributive law for arithmetic says that for any numbers x, y and z, $z(x+y) = zx+zy$. Use mathematical induction to show that the distributive law is valid for any number of terms in the sum. That is, show that for any integer $n \geq 2$, it is true that for any numbers z, x_1, x_2, ..., x_n,

$$z(x_1 + x_2 + \cdots + x_n) = zx_1 + zx_2 + \cdots + zx_n$$

(Use induction on n.)

3. The following funny recursive Java method finds the maximum value among the n array elements `A[0]`, `A[1]`, ..., `A[n]`, where $n \geq 0$:

```
static double recursive_max( double[] A, int n ) {
    if (n == 0) {
        return A[0];
    }
    else {
        int max1 = recursive_max(A, n-1);
        if (max1 > A[n])
            return max1;
        else
            return A[n];
    }
}
```

Use mathematical induction to prove that this algorithm does in fact return the largest value among `A[0]`, `A[1]`, ..., `A[n]`, for any integer $n \geq 0$.

4. (Exercise 2.1.7 from the textbook.) In the English sentence, "She likes men who are tall, dark, and handsome," does she like an intersection or a union of sets of men? How about in the sentence, "She likes men who are tall, men who are dark, and men who are handsome"? Explain.

5. Identify the set $\mathbb{R} \setminus \mathbb{Q}$. Explain your answer.

6. Let A be the set of positive even integers, $A = \{2, 4, 6, 8, 10 \ldots\}$. Let B be the set of positive odd integers, $B = \{1, 3, 5, 7, 9, \ldots\}$. Let C be the set of primes, $C = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$. And finally, let D be the set of positive multiples of 3, $D = \{3, 6, 9, 12, 15, \ldots\}$. Find the following sets, describing each set both in English and in set notation:

a) $A \cup B$ **b)** $A \cap B$ **c)** $C \setminus A$ **d)** $B \cup C$ **e)** $A \cap D$