CPSC 229, Fall 2007 — Homework #6

This homework is due in class on Friday, October 19. Originally, we were scheduled to have a test on October 19. However, that test has now been postponed to Monday, October 22.

1. Let A and B be 32-bit integers, and let M be the 32-bit integer that is represented in hexadecimal as `0xFFFF0000`. Consider the computation

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C = (A & M) | (B & ~M)
```

Explain carefully how the value of C is derived from the values of A and B. Give at least one example, for a particular choice of A and B (with A and B expressed in hexadecimal).

2. Let $A = \{1, 2\}$ and let $B = \{a, b\}$ (where a and b are distinct). Write out the set $\mathcal{P}(A \times B)$.

3. Let A and B be sets. Suppose that $A \times B = B \times A$. Does this imply that $A = B$? Explain.

4. Consider the following functions. For each function, decide whether the function is one-to-one, whether it is onto, and whether it is a bijection. Justify your answers.

a) $g\colon \mathbb{Z} \to \mathbb{Z}$, defined by the formula $g(x) = 3x$

b) $f\colon \mathbb{R} \to \mathbb{R}$, defined by the formula $f(x) = 3x$

c) $m\colon \mathbb{N} \to \mathbb{N}$, defined by $m(n) = \begin{cases} n-1 & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$

5. Let A, B and C be sets and suppose that $|A| = 3$, $|B| = 7$ and $|C| = 5$. Compute the cardinality of each of the following sets (using Theorem 2.8):

a) $C \times C$ **b)** $A \times B \times C$ **c)** $\mathcal{P}(B) \times \mathcal{P}(A)$ **d)** $\mathcal{P}(B \times A)$

6. Suppose that A and B are countably infinite sets. Show that $A \cup B$ is countably infinite.

7. Let Σ be the alphabet $\Sigma = \{a, b\}$, and define languages L and M over Σ as follows:

$$L = \{\varepsilon, a, aa, aaa\} \qquad M = \{b, bb, bbb, bbbb, \dots\}$$

Find each of the following languages:

a) L^2 **b)** M^2 **c)** LM **d)** $L \cup M$ **e)** $L \cap M$ **f)** L^*