This homework is due in class on Friday, November 16.

1. Give an English language description of the language generated by each of the following regular expressions over the alphabet $\{a, b, c\}$.

a) $(a+b)^*c(a+b)^*$
b) $(a+b+c)^*b(a+b+c)^*c(a+b+c)^*$
c) $(a+b+c)b(a+b+c)^*c(a+b+c)$

2. Give an argument to show that if L is a regular language, then L^R is also a regular language. (Hint: Start with a DFA that accepts L, and think about how you can produce a machine (either a DFA *or* and NFA) that accepts L^R.) You do not have to give a completely rigorous proof, but you should give a convincing argument.

3. Consider the following alphabet, in which each symbol is a stack of three 0's and 1's:

$$\Sigma = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$$

The addition of two binary numbers can be represented as a string over this Σ, with each symbol in the string representing one of the columns in the addition. For example,

$$\begin{array}{r} 00100101 \\ \underline{01001011} \\ 01110000 \end{array} \quad \text{is represented as} \quad \begin{bmatrix}0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\end{bmatrix} \begin{bmatrix}1\\0\\1\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\end{bmatrix}$$

A string over Σ, interpreted as an addition of binary numbers can be either a *correct* addition or an *incorrect* addition. Let L be the language over Σ that consists of all strings that represent correct additions. Show that this language is regular by constructing a DFA that accepts the language. If you prefer, you can show that L^R (and therefor also L) is regular by constructing a DFA that accepts L^R — it is actually easier to thing about the problem when the DFA reads the string from right-to-left. This exercise shows that the problem of checking whether a binary addition is correct is one that can be solved by a DFA.

4. Use the pumping lemma to prove that the language $L_1 = \{a^n b^m c^k \mid k = n + m\}$ is not regular.

5. Use the pumping lemma to prove that the language $L_2 = \{w \in \{a,b\}^* \mid w^R = w\}$ is not regular.

6. Use the pumping lemma to prove that the language $L_3 = \{a^{2^n} \mid n \in \mathbb{N}\}$ is not regular.