

*This is the homework for the week of September 1–5, covering Chapter 1, Sections 1 to 3. It is due in class on Wednesday, September 10. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.*

- Construct a truth table for each of the following compound propositions, and use it to determine whether the proposition is a tautology:

$$\text{a) } ((p \vee q) \wedge ((\neg p) \vee r)) \rightarrow (q \vee r) \qquad \text{b) } (p \rightarrow (q \vee r)) \rightarrow (p \rightarrow q)$$

- Draw a logic circuit that computes the value of the following propositions from its three inputs,  $A$ ,  $B$ , and  $C$ . Describe in words how you constructed the circuit

$$(A \wedge (B \vee \neg C)) \vee ((B \wedge C) \vee (A \wedge C))$$

- Draw a logic circuit with three inputs and one output that has the property that the output is *on* if and only if **exactly two** of the inputs are *on*. Explain your reasoning.
- Convert each of the following English statements into propositional logic. You should introduce symbols such as  $p$  and  $q$  to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for. The first statement is *ambiguous*; you should give two possible translations and explain the difference.

a) I like pizza with mushrooms and ham or sausage.

b) If Fred is a computer science major, then he must take CS229.

- Express the logical negation of each of the following sentences in natural English

a) There is no joy in Mudville.

b) The answer is either more than ten or less than three.

c) If Fred lives to be 100, he will still be a grouch.

- State the *converse* and the *contrapositive* of the English statement, “If it’s January, then it’s snowing in Geneva.” Explain in your own words why this statement and its converse are not logically equivalent.
- $\vee$ ,  $\wedge$ , and  $\rightarrow$  are examples of logical operators that act on two propositions. How many logical operators of this type are possible? Why? Hint: If a logical operator on two propositions is denoted by  $?$ , then it is completely defined by a truth table of the form

$p$	$q$	$p ? q$
T	T	
T	F	
F	T	
F	F	

How many truth tables are there of this form?

- Consider an ordinary deck of 52 playing cards. For how many cards in the deck is it true
  - that “This card is both a Jack and a Diamond”?
  - that “This card is either a Jack or a Diamond”?
  - that “This card is a Jack if and only if it is a Diamond”?
  - that “If this card is a Jack, then it is a Diamond”?
  - that “If this card is an Ace, then it is a Jack”?