*This is the homework for the week of September 8–12, covering Chapter 1, Sections 4 and 5. It is due in class on Wednesday, September 17. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.*

**1.** Find the negation of each of the following predicate logic expressions. Simplify your answers so that the negation operator, $\neg$, is applied only to simple terms like $P(x)$ and $L(z, y)$:

**a)** $\exists x\big(P(x) \wedge \neg Q(x)\big)$

**b)** $\forall x\, \exists y\big(\neg(P(y) \rightarrow K(y, x)\big)$

**c)** $\big(\exists x(\neg(P(x) \wedge Q(x)))\big) \wedge \big(\forall x(P(x) \vee Q(x))\big)$

**d)** $\exists r\, \exists s\, \big(P(r) \wedge P(s) \wedge (r \neq s) \wedge \big(\forall t(P(t) \rightarrow (t = s \vee t = r))\big)\big)$

**2.** Translate each of the following English sentences into predicate logic. Make up any predicates that you need, and specify what each predicate means. Also, state the domain of discourse of each predicate.

**a)** There is no joy in Mudville.

**b)** Every integer that is a multiple of 10 is also a multiple of 5.

**c)** For any integer, there is always a bigger integer.

**d)** Some pink elephants live in trees, but none live in purple houses.

**e)** Each Almond Joy has two almonds on top.

**3.** The sentence, "Someone has the answer to every question," is ambiguous. Give *two* translations of this sentence into predicate logic, and explain the difference in meaning.

**4.** Suppose that $S(p, m)$ means "$p$ has seen $m$," where the domain of discourse for $p$ is people, and the domain of discourse for $m$ is movies. State the meaning of each of the following propositions as an unambiguous English sentence:

**a)** $\exists p\, \exists m(S(p, m))$ **b)** $\forall p\, \exists m(S(p, m))$ **c)** $\exists m\, \forall p(S(p, m))$

**d)** $\forall p\, \forall m(S(p, m))$ **e)** $\exists p\, \forall m(S(p, m))$ **f)** $\forall m\, \exists p(S(p, m))$

**5.** Use a *truth table* to decide whether the following argument is valid:

$$\begin{array}{l} p \rightarrow q \\ (\neg p) \rightarrow q \\ \hline \therefore\ q \end{array}$$

**6.** Give a formal proof for each of the following valid arguments, includings the justification for each step in the proof:

**a)**
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow (r \vee s) \\ \neg s \\ p \\ \hline \therefore\ r \end{array}$$

**b)**
$$\begin{array}{l} (p \wedge q) \rightarrow (r \vee s) \\ \neg r \\ p \rightarrow q \\ p \\ \hline \therefore\ s \end{array}$$

**c)**
$$\begin{array}{l} p \rightarrow r \\ (r \wedge s) \rightarrow t \\ q \rightarrow \neg t \\ s \\ q \\ \hline \therefore\ \neg p \end{array}$$

**7.** Translate each of the following informal English arguments into informal argument by assigning propositional variables to represent the basic statements used in the arguments. Then decide whether or not the argument is valid. If the argument is valid, give a formal proof. If it is not valid, explain why.

**a)** If McCain is elected, the economy collapses. If Jack doesn't vote, then McCain is elected. Jack votes. So, the economy doesn't collapse.

**b)** In order to get a B.S. degree, you must pass a math class or a computer science class. If you don't understand algebra, you can't pass a math class. Mary has a B.S. degree, but Mary doesn't understand algebra. So Mary must have taken a computer science class.