CPSC 229, Fall 2008 — Homework #3

For the proofs that you are asked to do in this homework, you can give informal (but careful) proofs of the type that are typically given by mathematicians. You are not *required to give formal proofs.*

This homework is due in class on Wednesday, September 24. Remember that you can work with other people in the class, but you should write up your own solutions to turn in.

1. Suppose that a, b, and c are integers and that a is divisible by c. Prove that the product ab is also divisible by c.

2. Prove that for any integer n, the number $n(n+1)(n+2)$ is divisible by 3. (Consider giving a proof by cases. Note: it is a fact that any integer n can be written in one of the forms $3k$ or $3k+1$ or $3k+2$, where k is an integer. You can use this fact without proving it. You can also use problem **1**.)

3. Prove or disprove:

a) For any integer n, if n is divisible by 9, then so is n^2.

b) For any integer n, if n^2 is divisible by 9, then so is n.

4. Suppose that x, y, and z are real numbers and that $x+y+z$ is greater than 15. Use a proof by contradiction to show that at least one of x, y, and z must be greater than 5.

5. The textbook shows that the sum of two rational numbers is also a rational number (page 53). Prove that the *product* of two rational numbers is also a rational number. That is, prove: If x and y are rational numbers, then the product xy is also a rational number.

6. Use a proof by contradiction to prove: If x is an irrational number and r is a non-zero rational number, then xr is an irrational number.

7. Prove or disprove:

a) If x is an irrational number, then x^2 is also irrational.

b) If x^2 is an irrational number, then x is also irrational. (Hint: Consider the contrapositive.)

8. Use a proof by induction to show that $1+3+5+\cdots+(2n-1)=n^2$ for any positive integer n. That is, the sum of the first n odd integers is n^2.

9. Use a proof by induction to prove the following generalization of DeMorgan's law for propositional logic: Suppose that $p_1, p_2, \ldots, p_n$ are propositions, where $n \geq 2$. Then $\neg(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv (\neg p_1) \wedge (\neg p_2) \wedge \cdots \wedge (\neg p_n)$. (Use induction on n.)