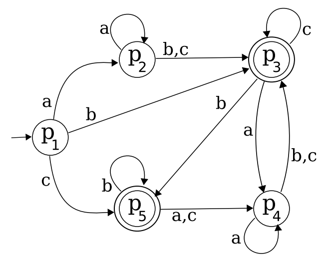
This homework is due in class on Friday, November 7.

1. A DFA, M, can be specified as a list of five things: $M = (Q, \Sigma, s, \delta, F)$. Consider the DFA that has the following transition diagram:



For this DFA, give the values of Q, Σ, s and F, and make a table for the transition function δ.

2. Write a regular expression for each of the following languages:

a) $L_1 = \{w \in \{a, b\}^* \,|\, w$ begins with ab and ends with $ba\}$

b) $L_2 = \{w \in \{a, b, c\}^* \,|\, w$ contains a b and there are no c's before the first b in $w\}$

c) $L_3 = \{w \in \{0, 1\}^* \,|\, n_1(w)$ is a multiple of $3\}$

d) $L_4 = \{w \in \{0, 1\}^* \,|\, n_1(w)$ is **not** a multiple of $3\}$

3. For each of the following languages, draw a DFA that accepts that language.

a) $L_2 = \{w \in \{a, b, c\}^* \,|\, w$ contains a b and there are no c's before the first b in $w\}$

b) $L_3 = \{w \in \{0, 1\}^* \,|\, n_1(w)$ is a multiple of $3\}$

c) $L_4 = \{w \in \{0, 1\}^* \,|\, n_1(w)$ is **not** a multiple of $3\}$

4. Suppose that M_1 and M_2 are DFAs over the same alphabet, Σ, where $M_1 = (Q, \Sigma, s, \gamma, F)$ and $M_2 = (P, \Sigma, t, \eta, E)$. Define the DFA M by $M = \big(Q \times P, \Sigma, (s, t), \delta, F \times E\big)$, where $\delta \colon (Q \times P) \times \Sigma \to (Q \times P)$ is given by $\delta((q, p), \sigma) = (\gamma(q, \sigma), \eta(p, \sigma))$. Note that states in M are ordered pairs (q, p) where q is a state in M_1 and p is a state in M_2, and that the start state of M is (s, t), where s is the start state of M_1 and t is the start state of M_2. What happens when M reads a string $w \in \Sigma^*$? Explain why M is essentially just "M_1 and M_2 running in parallel." Explain why $L(M) = L(M_1) \cap L(M_2)$. (You can give a formal proof of this if you want. It's actually not hard. Just consider $\delta^*((s, t), \sigma)$, which is used to define $L(M)$.)

5. Draw two DFAs, one for the language $M_1 = \{w \in \{a, b\}^* \,|\, w$ begins with $ab\}$ and another for the language $M_2 = \{w \in \{a, b\}^* \,|\, w$ ends with $ba\}$. Label the states in M_1 as $p_0, p_1, \ldots,$ and label the states in M_2 as $q_0, q_1, \ldots$ Then combine them into a DFA M constructed as in the previous problem, with states that are ordered pairs of states from M_1 and M_2. Label each state in M with an ordered pair (p_i, q_j). What language is accepted by M?