*This is the homework for the week of August 30 through September 3, covering Chapter 1, Sections 1 to 3. It is due in class on Wednesday, September 8. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in.* ***Show your work and explain your reasoning.*** *Remember that unsupported answers will receive little or no credit. You are encouraged to come in for help on doing the homework and on understanding the material, if you need it.*

**1.** Construct a truth table to show the following logical equivalence is valid:

$$(p \wedge q) \rightarrow \neg r \equiv \neg(p \wedge q \wedge r)$$

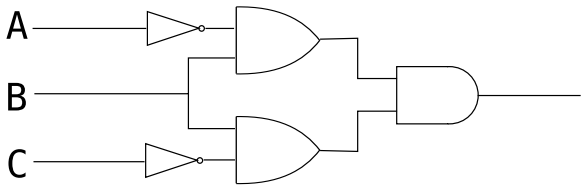
**2.** Construct a truth table to show that the following proposition is a tautology. Then explain in words why it *makes sense* for the proposition to be a tautology.

$$\big((p \vee q) \rightarrow r\big) \rightarrow (q \rightarrow r)$$

**3.** Draw a logic circuit that computes the value of the Boolean expression

$$(A \vee B \vee \neg C) \wedge \neg(A \wedge B \wedge C)$$

**4.** Consider the following logic circuit:




**a)** Write the Boolean algebra formula that expresses the output of this circuit in terms of the inputs, $A$, $B$, and $C$.

**b)** Simplify the formula from part **a)** by applying laws of Boolean algebra. For each step in the simplification, state which law you are applying. Try to find a formula that uses as few operators as possible.

**c)** Based on the simplified formula in part **b)**, draw a circuit that is simpler than the given circuit but is equivalent to it.

**5.** Draw a logic circuit with three inputs and one output that has the property that the output is *on* if and only if **exactly two** of the inputs are *on*. Explain your reasoning.

**6.** Convert each of the following English statements into propositional logic. You should introduce symbols such as $p$ and $q$ to stand for the simple propositions that occur in the statements. State clearly what each symbol stands for. Try to capture as much of the meaning of the sentence as you can.

**a)** If you are lucky, you will be rich and famous.

**b)** Betelgeuse is a star that is in the constellation Orion.

**c)** I like pizza with mushrooms or pepperoni.

**7.** Express the logical negation of each of the following sentences in natural, unambiguous English.

**a)** CPSC 229 is not meant to be fun.

**b)** The answer is greater than ten and less than twenty.

**c)** If Fred lives to be 100, he will still be a grouch.

**8.** Consider the statement, "If Casey strikes out, then there is no joy in Mudville." Express in natural English **a)** the *converse*, **b)** the *contrapositive*, and **c)** the *negation* of this statement.

**9.** Show that the following propositions are logically equivalent by finding a chain of logical equivalences from the first to the second. For each equivalence in the chain state what fact or law of Boolean algebra you are using.

$$p \to (q \to r) \qquad \text{and} \qquad (p \wedge q) \to r$$

**10.** Consider an ordinary poker deck of 52 playing cards. For how many cards in the deck is it true

**a)** that "This card is either a Heart or a Spade"?

**b)** that "This card is both an Ace and a Spade"?

**c)** that "This card is either a King or a Diamond"?

**d)** that "This card is a King if and only if it is a Heart"?

**e)** that "This card is a King if and only if it is not a Heart"?

**f)** that "If this card is a Queen, then it is a Diamond"?

**g)** that "If this card is an Ace, then it is a King"?