

This is the homework for the week of September 6–10, covering Chapter 1, Sections 4 and 5. It is due in class on Wednesday, September 15. Remember that you can work with other people in the class, but you should write up your solutions in your own words to turn in. Unsupported answers will not receive much credit!

- Write the *negation* of each of the following statements in simplified form. In your answer, the *not* operator, $\neg$, should only be applied to simple terms, such as $\neg P(a)$ or $\neg L(x, y)$.
 - $(\forall x (P(x))) \vee (\forall x (Q(x)))$
 - $\forall x (P(x) \vee Q(x))$
 - $\exists x \forall y (L(y, x) \rightarrow P(y))$
 - $\forall n (Z(n) \rightarrow \exists k (Z(k) \wedge G(k, n)))$
- Explain the difference in meaning between parts **a)** and **b)** in the preceding problem, and explain the relationship between them. (One of them implies the other.)
- Translate each of the following English sentences into predicate logic. Make up any predicates that you need. State what each predicate means and what its domain of discourse is. Try to capture as much of the meaning of the English statement as you can.
 - All cats are selfish.
 - Some politician is honest.
 - There is a book that no one has read.
 - Everyone who owns a yacht is rich.
 - Some problem has more than one solution.
- The sentence “Someone has the answer to every question” is ambiguous. Give two translations of this statement into predicate logic, and carefully explain the difference in meaning.
- Give a formal proof of that each of the following arguments is valid. State the justification for each step in the proof.

$ \begin{array}{l} \text{a) } (p \wedge \neg q) \rightarrow r \\ q \rightarrow \neg p \\ p \\ \hline \therefore r \end{array} $	$ \begin{array}{l} \text{b) } p \rightarrow r \\ (r \wedge s) \rightarrow t \\ q \rightarrow \neg t \\ s \\ q \\ \hline \therefore \neg p \end{array} $	$ \begin{array}{l} \text{c) } (r \wedge s) \rightarrow p \\ \neg(p \wedge q) \\ r \\ q \\ \hline \therefore \neg s \end{array} $
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- Translate each of the following arguments, expressed in English, into logic, and determine whether the argument is valid. If the argument is valid, give a formal proof. If it is not valid, explain why.

- a) If this card is red, then it's the King of Hearts. But if this card is a King, then it is not a Heart. So, this card is not red.
- b) A math major takes Abstract Algebra or Foundations of Analysis. If one takes Abstract Algebra, one knows about Galois. Mary graduated with a major in math, but she never took Foundations of Analysis. So, Mary knows about Galois.
- c) If Jack stays up late partying, he is tired the next day. If Jack is tired and there is a test, he doesn't do well on the test. If Jack does well on a test, he celebrates. There was a test today, and Jack is not celebrating, so he must have stayed up late partying last night.