This homework covers mathematical induction and the beginning of Chapter 2. It is due in class on Monday, October 4. Remember that you can work with other people in the class, but you should write up your solutions in your own words to turn in. For full credit, you must show your work and present careful and well-organized proofs.

1. (This is Exercise 1.9.2.) Consider the Towers Of Hanoi method from Section 1.9:

```
static void Hanoi(int n, int A, int B, int C) {
   if ( n == 1 )
      System.out.println( "Move disk 1 from " + A + " to " + B );
   else {
      Hanoi( n-1, A, C, B );
      System.out.println( "Move disk " + n + " from " + A + " to " + B);
      Hanoi( n-1, C, B, A );
   }
}
```

Use mathematical induction to prove that for any integer $n \geq 1$, the number of lines of output produced by a call to `Hanoi(n,a,b,c)` is $2^n - 1$. (Note that the parameters a, b, and c are irrelevant to this question.)

2. DeMorgan's Law for $\wedge$ is a statement about the negation of the conjunction of *two* propositions: $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$. Use mathematical induction to show that it actually holds for n propositions for any integer $n \geq 2$. That is, prove that for any $n \geq 2$, it is true that for any propositions $p_1, p_2, \dots, p_n$,

$$\neg(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \equiv (\neg p_1 \vee \neg p_2 \vee \cdots \neg p_n)$$

3. Here is a particularly pretty result that I found on the Web; it provides another proof that the number of prime numbers is infinite: Prove by induction that for any integer $n \geq 1$, the number $2^{2^n} - 1$ is divisible by at least n different prime numbers. [Hints: $2^{2^n} - 1 = \big(2^{2^{n-1}} - 1\big) * \big(2^{2^{n-1}} + 1\big)$. $2^{2^{n-1}} + 1 = \big(2^{2^{n-1}} - 1\big) + 2$. Any integer greater than 1 is divisible by at least one prime. And $2^{2^{n-1}} - 1$ is an odd number and hence not divisible by 2.]

4. Let $A = \{1, 3, 5\}$, let $B = \{2, 4, 5, 6\}$, let $C = \{\{1\}, \{2, 3\}\}$, and let $D = \{\{1, 3\}, \{2, 3\}\}$. Compute each of the following sets:

a) $A \cup B$ **b)** $A \cup C$ **c)** $A \cap C$ **d)** $C \cap D$
e) $B \smallsetminus A$ **f)** $A \smallsetminus C$ **g)** $C \smallsetminus D$ **h)** $\mathcal{P}(D)$

(For this problem only, you do not have to justify your work.)

5. Identify the set $\mathbb{R} \smallsetminus \mathbb{Q}$. Explain your answer.

6. Let A, B, and C be sets. Suppose that $A \subseteq C$ and $B \subseteq C$. Prove that $A \cup B \subseteq C$.

7. Use the laws of logic to verify that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ for all sets A and B.

8. (Exercise 2.1.7 from the textbook.) In the English sentence, "She likes men who are tall, dark, and handsome," does she like an intersection or a union of sets of men? How about in the sentence, "She likes men who are tall, men who are dark, and men who are handsome"? Explain.