This homework covers selected material from Sections 2.3, 2.4, 2.6, and 3.1. It is due in class on Monday, October 18. The second test of the semester will be given on Wednesday, October 20.

1. Let m and n be the 16-bit binary numbers shown below. Compute the 16-bit binary numbers $n \,\&\, m$, $n \mid m$, and $\tilde{}m$, and write out the subset of $\{0, 1, 2, \dots, 15\}$ that is represented by each of the numbers m, n, $n \,\&\, m$, $n \mid m$, and $\tilde{}m$.

$$n = \texttt{1011 0010 1010 0011}$$
$$m = \texttt{0001 1000 1110 0110}$$

2. Let A and B be 32-bit integers, and let M be the 32-bit integer that is given in hexadecimal as `0xFFFF0000`. Consider the assignment statement:

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C = (A & M) | (B & ~M);
```

Carefully explain how the value of C is obtained from the values of A and B. Give at least one example, with specific values for A and B, with A and B expressed in hexadecimal.

3. Let x and y be binary numbers. Depending on the bit-patterns, the numbers $x \mid y$ and $x + y$ might or might not be the same. Determine the circumstances under which they will be the same. Justify your answer.

4. Suppose A, B, and C are finite sets and that $|A| = 12$, $|B| = 5$, and $|C| = 7$. Also assume that $A \cap B = \emptyset$. Find the cardinality of each of the following sets. Use Theorem 2.8.

a) $A \times B$ **b)** $A \cup B$ **c)** $\mathcal{P}(B) \times \mathcal{P}(C)$ **d)** $\mathcal{P}(B \times C)$

e) Why can't I ask you to find $|B \cup C|$?

5. Let X and Y be sets. Suppose that the sets $X \times Y$ and $Y \times X$ are equal. Does it necessarily follow that $X = Y$? Justify your answer.

6. Let X be an uncountable set, and let C be a countably infinite subset of X. Show that $X \smallsetminus C$ is uncountable. (Hint: Use proof by contradiction. We showed in class that the union of two countably infinite sets is countably infinite.) What does this say about the set of irrational numbers? Why?

7. Consider the alphabet $\Sigma = \{a, b, c\}$. Let K, L, and M be the languages over M given by:

$$K = \{\varepsilon, a, b, c\} \qquad L = \{aa, ab\} \qquad M = \{c, cb, cbb, cbbb, cbbbb, \dots\}$$

Find the following languages. In each case, list the elements of the language or give a clear description of the set of strings that make up the language.

a) $K \cup L$ **b)** KL **c)** LK **d)** KK **e)** LLL

f) L^* **g)** LM **h)** MM **i)** M^* **j)** K^*

8. We defined the reverse of a string $x = a_1 a_2 \cdots a_k$ to be $x^R = a_k a_{k-1} \cdots a_1$. The reverse of a **language** T is defined as $T^R = \{x^R \mid x \in T\}$. Suppose that K, L, and M are the languages given in the preceding problem. Find K^R, L^R, and M^R.