This homework covers Sections 3.7 and 4.1 (with question 1 as a bonus). It is due on Monday, November 15. There

1. Consider the following alphabet, in which each symbol is a stack of three 0's and 1's:

$$\sum \;=\; \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$$

The addition of two binary numbers can be represented as a string over this Σ, with each symbol in the string representing one of the columns in the addition. For example,

$$\begin{array}{r} 00100101 \\ \underline{01001011} \\ 01110000 \end{array} \quad \text{is represented as} \quad \begin{bmatrix}0\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\end{bmatrix} \begin{bmatrix}1\\0\\1\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\end{bmatrix}$$

A string over Σ, interpreted as an addition of binary numbers can be either a *correct* addition or an *incorrect* addition. Let L be the language over Σ that consists of all strings that represent correct additions. Show that this language is regular by constructing a DFA that accepts the language. If you prefer, you can show that L^R (and therefor also L) is regular by constructing a DFA that accepts L^R — it is actually easier to think about the problem when the DFA reads the string from right-to-left. This exercise shows that the problem of checking whether a binary addition is correct is one that can be solved by a DFA. **(This problem can be solved with a DFA that has just four states (or three states, if you consider the empty string to be acceptable). "Carries" from one column to the next play a major role in the design.)**

2. Use the Pumping Lemma to prove that the language $\{a^n b^m c^k \mid n + m = k\}$ is not regular.

3. Use the Pumping Lemma to prove that the language $\{w \in \{0,1\}^* \mid w = w^R\}$ is not regular.

4. The Pumping Lemma says that for any regular language L, there is an integer K such that for any $w \in L$ such that $|w| \geq L + K$, w can be written $w = xyz$, where $y \neq \varepsilon$ and $xy^n z \in L$ for all $n \in \mathbb{N}$. It looks like this makes it possible to produce an infinite number of strings in any regular language L, namely xz, xyz, $xyyz$, $xyyyz$, $\ldots$. But any *finite* language is regular. If L is a finite language, it can't contain an infinite number of strings! What actually happens with the Pumping Lemma in this case? (The answer is very simple and short.)

5. For each of the following Context-Free Grammars, find the language generated by the grammar. Briefly justify your answers.

a)
$S \longrightarrow aT$
$T \longrightarrow aTb$
$T \longrightarrow \varepsilon$

b)
$S \longrightarrow TaT$
$T \longrightarrow TT$
$T \longrightarrow aTb$
$T \longrightarrow bTa$
$T \longrightarrow \varepsilon$

c)
$S \longrightarrow aSc$
$S \longrightarrow R$
$R \longrightarrow bRc$
$R \longrightarrow \varepsilon$

6. For each of the following languages, create a Context-Free Grammar that generates that language. Explain in words why your grammar works.

a) $\{a^n b a^n \mid n \in \mathbb{N}\}$ **b)** $\{a^n b^m c^k \mid m > n + k\}$ **c)** $\{a^n b^m c^k d^l \mid n + m = k + l\}$