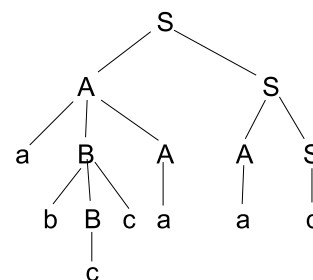


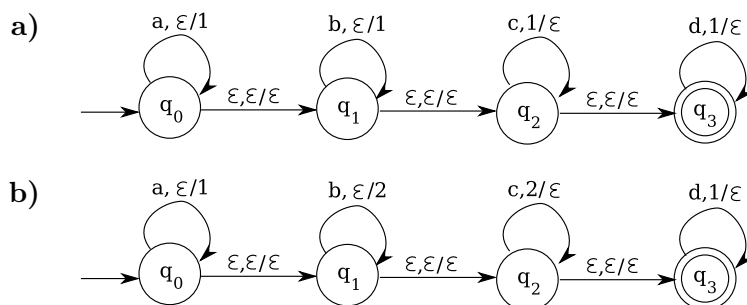
*This is the final homework assignment that will be collected and graded for this course. It covers Sections 4.3 through 4.6. It is due in class on Wednesday, December 8.*

1. Consider the parse tree shown at the right, which is based on a context-free grammar  $G$ ,



- Based on the fact that this is a valid parse tree, give five different production rules that must be part of the grammar  $G$ .
- Give the left derivation that corresponds to this parse tree.
- Give the right derivation that corresponds to this parse tree.

2. For each of the following pushdown automata, determine the language that is accepted by the automaton. Justify your answer by describing how the automaton works.



- Use the pumping lemma for context free languages to show that the language  $\{a^n b a^n b a^n \mid n \in \mathbb{N}\}$  is not context-free.
- Consider the grammar for the language  $\{a^{n^2} \mid n \in \mathbb{N}\}$  given in the textbook on page 243. Give a derivation of the string  $aaaa$  using that grammar.
- Find a general grammar that generates the language  $\{w \in \{a, b, c\}^* \mid n_a(w) > n_b(w) > n_c(w)\}$ . (Compare to the language  $\{w \in \{a, b, c\}^* \mid n_a(w) = n_b(w) = n_c(w)\}$ , which was done in class. Explain how your grammar works.
- Find a general grammar that generates the language  $\{a^n b a^n b a^n \mid n \in \mathbb{N}\}$ . (Compare to the language  $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ , which was done in class.) Explain how your grammar works.
- Find a general grammar for the language  $\{w w \mid w \in \{a, b, c\}^*\}$ . (A similar language over the alphabet  $\{a, b\}$  was done in class.) Explain how your grammar works.
- Find a general grammar for the language  $\{a^n b^k c^{nk} \mid n, k \in \mathbb{N}\}$ . Explain how your grammar works.
- Draw a transition diagram for a Turing Machine that decides the language  $\{w \in \{a, b\}^* \mid n_a(w) \text{ is a multiple of } 3\}$ .