The second test for this course will be given in class on Wednesday, October 20. It covers everything that we have done from Chapter 2, Sections 1, 2, 3, 4, and 6; it also covers Section 3.1. Note that we skipped some of the material in the listed sections in Chapter 2. As usual, the test will include some "essay-type" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework. This might include some proofs. There might be some questions related to the programming assignment and the various ways of representing sets in Java.

Here are some terms and ideas that you should be familiar with for the test:

sets
set notations: $\{a, b, c\}$, $\{1, 2, 3, \dots\}$, $\{x \mid P(x)\}$, $\{x \in A \mid P(x)\}$
the empty set, $\emptyset$ or $\{\,\}$
equality of sets: $A = B$ iff they contain the same elements
element of a set: $a \in A$
sets can contain other sets as elements
subset: $A \subseteq B$
$A = B$ if and only if both $A \subseteq B$ and $B \subseteq A$
proper subset
union, intersection, and set difference: $A \cup B$, $A \cap B$, $A \smallsetminus B$
definition of set operations in terms of logical operators
disjoint sets ($A \cap B = \emptyset$)
power set of a set: $\mathcal{P}(A)$
universal set
complement of a set (in a universal set): $\overline{A}$
DeMorgan's Laws for sets
Russell's Paradox
there is no "set of all sets"
bitwise operations in Java: `&, |, ~`
using an n-bit integer to represent subsets of $\{0, 1, 2, \dots, n-1\}$
`&, |`, and `~` as set operations (intersection, union, complement)
the shift operators `<<`, `>>`, and `>>>`
using "`1 << n`" to represent the singleton set $\{n\}$
representing a set of integers as a boolean array in Java
Java's `TreeSet<Integer>` class; operations `set.add(n)`, `set.remove(n)`, `set.contains(n)`
ordered pair: (a, b)

equality of ordered pairs; $(a, b) = (c, d)$ iff $a = c$ and $b = d$

cross product of sets: $A \times B$

one-to-one correspondence

cardinality of a finite set: $|A|$

cardinality rules for finite sets: $|A \times B| = |A| \cdot |B|$, $|\mathcal{P}(A)| = 2^{|A|}$, $|A \cup B| = |A| + |B| - |A \cap B|$

finite set (in one-to one correspondence with one of the sets $N_0, N_1, N_2, \ldots$)

infinite set (not finite)

countably infinite set (in one-to-one correspondence with $\mathbb{N}$)

a set is countably infinite iff its elements can be placed into an infinite list

countable set (finite or countably infinite)

uncountable set (that is, uncountably infinite)

Cantor's dialgonalization proof that the set of real numbers is uncountable

examples of countably infinite and uncountably infinite sets

the union of two countably infinite sets is countably infinite

the cross product of two countably infinite sets is countably infinite

if X is uncountable and A is a countable subset, then $X \smallsetminus A$ is uncountable

for any set A, there is no one-to-one correspondence between A and $\mathcal{P}(A)$

proof of the above fact (given $f \colon A \to \mathcal{P}(A)$, consider $X = \{a \in A \mid a \notin f(a)\}$)

the power set of a countably infinite set is uncountable

function $f \colon A \to B$

a function $f \colon A \to B$ as a subset of $A \times B$

alphabet (finite, non-empty set of "symbols")

string over an alphabet Σ

length of a string, $|x|$

empty string, ε

concatenation of strings, xy or $x \cdot y$

reverse of a string, x^R

x^n, for a string x and a natural number n

$n_\sigma(x)$, the number of occurrences of a symbol σ in a string x

the set of all possible strings over Σ, denoted Σ^*

language over an alphabet Σ (a subset of Σ^*)

a language over Σ is an element of $\mathcal{P}(\Sigma^*)$

the set of strings over Σ is countable; the set of languages over Σ is uncountable

operations on languages

union, intersection, set difference, and complement applied to languages

concatenation of two languages: LM

L^n, for a language L and a natural number n

Kleene star of a language: L^*