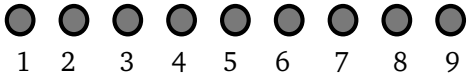
9 coins; one coin is counterfeit.
All real coins weigh the same.
The counterfeit weighs either
less or more than a real coin.



Problem: Find the counterfeit and whether it is lighter or heavier, using a balance just three times.

1,2,3 vs. 4,5,6

same

1,2,3 is lighter

4,5,6 is lighter

The counterfeit must be 7, 8, or 9.

The counterfeit is 1, 2 or 3 AND is lighter OR it is 4, 5, or 6 AND is heavier.

(Similar to previous case by symmetry.)

7 vs. 8

same

7 is lighter

8 is lighter

1,4 vs. 2,5

same

1,4 is lighter

2,5 is lighter

9 is the counterfeit

7 is bad AND is light OR 8 is bad AND is heavy

8 is bad AND is light OR 7 is bad AND is heavy

3 is bad AND is light OR 6 is bad AND is heavy

(Simlar to next case.)

1 is bad AND is light OR 5 is bad AND is heavy**

2 is bad AND is light OR 6 is bad AND is heavy

(Similar to previous case.)

9 vs. 1

7 vs. 1

(Similar to next case.)

8 vs. 1

1 vs. 7

9 is lighter

9 is heavier

8 is lighter

same

1 is lighter

same

9 is the counterfeit and is lighter

9 is the counterfeit and is heavier

8 is the counterfeit and is lighter

7 is the counterfeit and is heavier

1 is the counterfeit and is lighter

5 is the counterfeit and is heavier

(Note: 9 can't be the same weight as 1 because we know that 9 is the counterfeit.)

(Note: 8 can't be heavier than 1 because we know that if 8 is the counterfeit, then it is lighter than a real coin.)

**(Note: When we compare 1,4 to 2,5, if 1,4 is lighter than 2,5, then it is not possible for 4 to be the counterfeit. This is because if 4 is counterfeit, it is heavier than a real coin, so 1,4 would be heavier than 2,5, not lighter. Similalry, it's impossilbe for 2 to be the counterfeit, since if it were, it would be ligher than a real coin, and 1,4 would be heavier than 2,5.