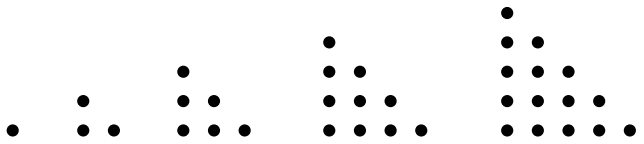
Instructions: This homework is due next Wednesday, February 5. You will have some time in class to discuss this homework in small groups, and you can work with other students on it outside of class. However, you should write up your own answers in your own words to turn in. Remember that all answers should be neatly written, should show your work, and should usually include explanations in the form of regular sentences and paragraphs.

1. Write down the first 7 numbers in the sequence defined by $\begin{cases} A_1 = 1 \\ A_n = (2 \times A_{n-1}) + 1, & \text{for } n > 1 \end{cases}$

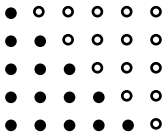
2. (Triangular Numbers) We saw in class how a "square number" can be represented as some dots arranged in the shape of a square. Triangular numbers are made by arranging dots into the shape of a triangle. Here is a picture of the first five triangular numbers:



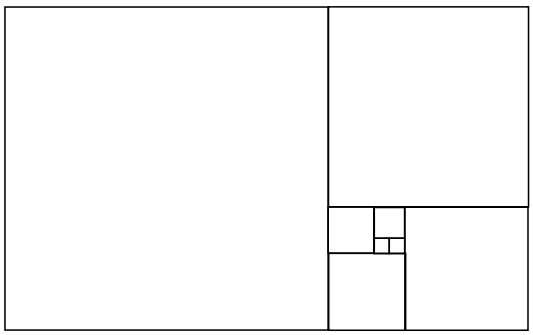
a) Write down the first 8 triangular numbers. Explain how you got the numbers that come after the 5 that are shown here.

b) Count the number of dots in each row of a triangular number. Based on the result, you can write the n^{th} triangular number as a simple sum of n numbers. What is the sum, and why?

c) Use the following picture to guess a simple formula, not a long sum, for the n^{th} triangular number. (Explain your reasoning!!) Hint: You should see two copies of some triangular number in the picture.



3. (A Fibonacci formula) We looked at the following picture in class. It shows a rectangle that is made up of squares, where the sizes of the squares are given by the Fibonacci sequence. We saw how you can keep adding new squares to get larger and larger rectangles. This question is about all the rectangles that you get in this way, not just the particular case shown here.



a) Suppose that the size of the largest square is F_n, the n^{th} Fibonacci number. Explain why the area of the entire rectangle is $F_n * F_{n+1}$.

b) You can also get the area of the rectangle by adding up the areas of the individual squares that make up the rectangle. In this way, you can express $F_n * F_{n+1}$ as a certain sum. Write down the sum for the case $n = 8$ and check that the value is indeed equal to $F_8 * F_9$. (Explain how the sum is obtained from the picture!!) You should try to find a formula that will work for all n, not just for the specific case $n = 8$.

4. (Fibonacci-like sequences) For this problem, use the web page *NumberSeq.html* to do the computations that you need. You can find a link to it on the class web page, or go directly to this web address:

`math.hws.edu/eck/math110/s14/NumberSeq.html`

This page will compute sequences A_n that are defined by rules similar to the rules for the Fibonacci sequence. It also shows the quotients A_n/A_{n-1} of each term divided by the previous term. You get to input the values for A_1 and A_2. You can also input numbers m_1 and m_2 to be used in the rule $A_n = (m_1 \times A_{n-1}) + (m_2 * A_{n-2})$. (For the Fibonacci sequence, m_1 and m_2 are both equal to 1.)

a) The Lucas numbers L_n are given by $L_1 = 2$, $L_2 = 1$, and $L_n = L_{n-1} + L_{n-2}$ for $n > 2$. Find the first 10 Lucas numbers. (Use the web page.) What appears to happen to L_n/L_{n-1} as n gets bigger?

b) Try some other values for A_1 and A_2, but leave m_1 and m_2 set equal to 1. Use positive integers for A_1 and A_2. What happens to A_n/A_{n-1} as n gets bigger? Make a *conjecture* about what happens. (You are being asked to make a guess, not prove it.)

c) Now, try varying m_1 and m_2 as well as A_1 and A_2, and observe A_n/A_{n-1} as n gets bigger. Describe what happens and make a conjecture about what happens in general.

5. Consider the pair of numbers n and $n + 1$.

a) Find an integer n such that both n and $n + 1$ are prime numbers.

b) Show that the n that you find for part **a)** is the *only* possible answer. (That is, show that for any *other* value of n, it can't be the case that n and $n + 1$ are both prime.)

6. You are in a room with 100 politicians. You know that (a) At least one of the politicians is honest; and (b) If you talk to *any* two of the politicians in the room, at least one of the two is dishonest. Exactly how many dishonest politicians are in the room? Explain!