Math 110-02, 26 March 2014 **Name:**

Instructions: *Work on this assignmnet in class with a group of two or three people. A few people will present their results before the end of the class period. Finish the assignment, writing things up in your own words, and turn it in on Wednesday. This will be graded on a check/check-plus/check-minus basis.*

Exercise 1. Two Circles. This is Exercise 3.5.6 from page 200 of the textbook, which has a hint in the back of the book. Prove that a small circle has the same number of points as a large circle. Stated precisely, prove that there cardinalities are the same by describing a one-to-one correspondence between the points on the small circle and the points on the large circle. (Start by drawing the small circle *inside* the large circle.)

Exercise 2. An extra point. We have seen that any two open intervals of real numbers have the same cardinality. But suppose we compare the open interval from 0 to 1, excluding both endpoints, to the "half-open" interval from 0 to 1 that includes 1 but does not include 0. Mathematicians use the notations $(0, 1)$ and $(0, 1]$ for these intervals. It seems like $(0, 1]$ must have the same cardinality as $(0, 1)$, but to prove it we must find an actual one-to-one correspondence between them.

Find a one-to-one correspondence from the half-open interval $(0, 1]$ to the open interval $(0, 1)$. Here's a hint: Almost every point in $(0, 1]$ can be matched up with the same point in $(0, 1)$. Other hints might be added in class.

Exercise 3. Two extra points. Now consider the closed interval $[-1, 1]$, which includes both endpoints. Find a one-to-one correspondence from $[-1, 1]$ to the open interval $(-1, 1)$. This should be easy for you, if you've done exercise 1.

Exercise 4. An extra circle. The closed unit disk consists of a circle of radius 1 and all the points inside the circle. The open unit disk consists of just the points inside the circle. The open unit disk is the closed unit disk with the boundary circle removed. Describe a one-to-one correspondence between the open unit disk and the closed unit disk. You don't have to specify an explicit mapping—just describe in words how the mapping would be done.