The first test in this course will be given in class on Friday, February 22. About half the test will be on the readings from *The Language Instinct*. You should expect essay questions that are longer and more in-depth than those given on the quiz. They will mostly require you to understand the general arguments given in the book, rather than a lot of the detailed facts that are presented.

About one-fourth of the test will be on voting. Most of the questions will be similar to the ones that were on the homework. You should understand the plurality, plurality with runoff, plurality with elimination, and Borda count voting methods. You should know about Condorcet winners and losers. You will not be asked about other voting methods or criteria, such as the Pareto or monotonicity conditions.

Finally, there will be some questions based on work that we did from *Problem Solving Through Recreational Mathematics*. You should understand enough about algebra to be able to give names to variables and write down equations. I will not give very difficult problems on the test, and I will not ask you to actually solve any equations. There will also be something on the test about solving logic puzzles, but again, you will not have to solve difficult puzzles. There is nothing about the first homework assignment (finding areas of flags).

Sample Solutions to Algebra Homework

3.3. Let x be the amount of money that still has to be raised. The goal of the problem is to find the value of x. Since the total amount that has to be raised is \$70,000, and x still needs to be raised, we can say that the amount that has been collected so far is $70,000 - x$ dollars. We know that one-third of the amount that has already been collected is equal to three-fifth of the amount still needed. This can be translated into the equation $\frac{1}{3}(70000 - x) = \frac{3}{5}x$. Multiplying this equation by 15 gives $5(70000 - x) = 9x$, or $350000 - 5x = 9x$, or $14x = 350000$, or $x = 25000$. That is, the amount that still has to be raised is \$25,000.

3.13 Let c be the number of cowboys (with their horses), and let a be the number of riderless animals. Note that the toll will then be $25c + 10a$ cents. We have to determine c and a. We are told that there are 4248 legs in total. Each riderless animal contributes 4 legs to this total, and each cowboy+horse contributes 6 legs. This can be expressed in the equation $4a + 6c = 4248$. Also, there are 1078 heads. A riderless animal has one head while a cowboy+horse has two. So, we get a second equation $a + 2c = 1078$. Solving the second equation for a gives $a = 1078 - 2c$. Plug this back into the first equation to get $4(1078 - 2c) + 6c = 4280$, or $4312 - 8c + 6c = 4248$, or $4312 - 2c = 4248$. Subtract 4280 from both sides and add $2c$ to get $64 = 2c$, or $c = 32$. Then, from $a = 1078 - 2c$, we get $a = 1078 - 64 = 1014$. There are 1014 riderless animals and 32 cowboys with horses. The toll is $25 \cdot 32 + 1014 \cdot 10$ cents. That's 10940 cents or \$109.40.

3.35 Think of "X" marking the spot where the two trains meet. They meet after traveling for 9 hours. After the meeting, the Miami-to-DC train takes another 6 hours to finish the trip from X to DC. That train will cover in 6 hours the same distance between X and Miami that the DC-to-Miami train has just taken 9 hours to cover. The DC-to-Miami train takes $1\frac{1}{2}$ times longer than the Miami-to-DC takes to cover the same distance. After the trains meet, the DC-to-Miami train still has to cover a certain distance, from X to Miami. That distance took 9 hours for the Miami-to-DC train; it will take $1\frac{1}{2}$ times as long as that for the DC-to-Miami train. So, the DC-to-Miami train still has another $13\frac{1}{2}$ hours to go after 5:00 PM until it gets to Miami. It will arrive there at 6:30 AM the next day.

3.47 One way to do this problem is to look at the total number of worker hours that it takes to paint the house. This can be computed as the number of workers times the number of hours that it takes those workers to do the job. Let w be the number of workers on the regular crew. We are told that the regular crew would take 8 hours to paint the house, so the number of worker hours is $8w$. Furthermore, if three extra workers are hired, the number of hours goes down to 6. If there are three more workers than on the regular crew, the number of workers is $w + 3$, and so the total number of worker hours to do the job is $6(w+3)$. The total number of worker hours must be the same no matter how many workers there are, so we must have $8w = 6(w+3)$, or $8w = 6w+18$, or $2w = 18$, or $w = 9$. So, 9 workers working together take 8 hours to do the job, for a total of 72 worker hours. If there is only one worker, that worker would take all 72 hours to complete the job.

3.50 To solve this problem, we need to figure out how many eggs, on average, one hen lays in one day. Let x represent this quantity. x can be computed as (number of eggs) divided by ((number of hens)$\times$(number of days)). Since $1\frac{1}{2}$ hens produce $1\frac{1}{2}$ eggs in $1\frac{1}{2}$ days, $x = (1\frac{1}{2})/(1\frac{1}{2} \cdot 1\frac{1}{2})$, which is $\frac{2}{3}$. That, one hen produces $\frac{2}{3}$ eggs per day on average. This means that in seven days, seven hens will produce on average $7 \cdot 7 \cdot \frac{2}{3}$ eggs. That's $\frac{98}{3}$, or $32\frac{2}{3}$, eggs.

Another way to find x is to note that if $1\frac{1}{2}$ hens produce $1\frac{1}{2}$ eggs in $1\frac{1}{2}$ days, then one hen working alone will produce one egg in $1\frac{1}{2}$ days. One day would be $\frac{2}{3}$ of the time span of $1\frac{1}{2}$ days, so in one day, one hen will produce $\frac{2}{3}$ of one egg.