This lab is to be turned in at next week's lab period, on November 16. Present your responses fully and clearly. Reminder: There is a test coming up next week, on Friday, November 19.

1. Consider the simple cubic polynomial function $f(x) = x^3 - 3x^2 - 9x + 2$. Find the critical points of f. Find the intervals on which f is increasing and on which it is decreasing. Find any local maxima and local minima of f. Find the intervals on which f is concave up and on which it is concave down. Find any inflection points of f.

2. Consider a quadratic polynomial function $f(x) = ax^2 + bx + c$ (where $a \neq 0$).

a) Show that f has one critical point and find it.

b) What can you say about the *concavity* of $f(x)$?

c) The critical point can be an absolute maximum or an absolute minimum. For which values of a is it a maximum point? For which values of a is it a minimum point? Why?

d) Suppose that the equation $ax^2 + bx + c = 0$ has two solutions, x_1 and x_2. Verify that the critical point of f is $(x_1 + x_2)/2$; that is, the critical point lies halfway between the two roots.

3. For this problem, you should investigate the maxima and minima of the functions $x^n e^{-x}$, for all positive integers n. Present your results as a short essay, with equations, calculations, and drawings included as appropriate. You can use a graphing calculator to look at some graphs, if you want.

Start by finding the critical points of $x^n e^{-x}$. Each of these functions has a local maximum. Find that maximum. For which values of n is the local maximum an absolute maximum? Discuss the pattern in words. (How does the maximum point depend on n?)

Some of the functions—about half of them—also have an absolute minimum point. For which values of n does the function have a minimum, and where is the minimum point?

Draw the graph of $y = x^n e^{-x}$ for a few representative values of n. (Note: all of these functions have the property that $\lim_{x\to\infty} x^n e^{-x} = 0$. We will be able to prove that after Section 4.7.)

4. For this problem, you should investigate the graphs $f(x) = mx + \sin(x)$, where m is a constant and $m > 0$. Present your results as a short essay, with equations, calculations, and drawings included as appropriate. You can use a graphing calculator to look at some graphs, if you want.

Describe how this function is created by adding a line to a sine function. Find the x-values where f has inflection points. Find any critical points of f. For which values of m do critical points exist? For which values of m does the function have local minima and maxima? For which **one** (positive) value of m does $f(x)$ have critical points that do **not** correspond to local minima and maxima? (Hint: For that value of m, the function is strictly increasing, but it "flattens out" and has inflection points at the critical points.) Sketch the graph for that value of m and for a couple other typical values.