This lab is mostly review for the test on Friday. ***Problems 1 through 3 will be collected for grading during the lab at 2:45,*** *and a solution sheet for the entire lab will be available at that time. The remaining problems are for practice and will not be collected or graded. (Problems 4 and 5 are there partly to show that there are some word problems that have reasonable computations and numbers!)*

1. Polynomial Graph. (Part **a** of this problem arose when I was trying to find a function for part **b** with the properties that I wanted.

a) Show that the equation $2x^3 - 3x^2 - 4 = 0$ has $x = 2$ as its only solution by verifying that $2x^3 - 4x^2 - 4$ can be factored as $(x-2)(2x^2 + x + 2)$ and that $2x^2 + x + 2$ has no real roots.

b) Graph the function $f(x) = \frac{1}{2}x^4 - x^3 - 4x$. Find the first derivative, and use it to find where f is increasing, where it is decreasing, and where it has a local minimum or maximum. Find the second derivative, and use it to find where f is concave up, where it is concave down, and where it has points of inflection. Be sure to find $f(x)$ at each local extremum and inflextion point (at least), and find $f'(x)$ at each inflection point (at least). Draw a careful graph that uses all the information that you have found. Present your work in an organized and clear format.

2. Graph with logarithmic function. Consider the function $f(x) = x\ln(x)$. Note that $f(x)$ is defined only for $x > 0$. Make a careful graph of f, using all the information that you can get from the first and second derivatives. It is not entirely clear *a priori* what happens to this function near $x = 0$. Compute $f(x)$ for several values of x near zero, and make an educated guess about $\lim_{x\to 0} f(x)$. (You can use your calculator.) Present your work in an organized and clear format.

3. Graph with asymptotes. Let $f(x) = \frac{1}{x}e^x$. Make a careful graph of $f(x)$. Compute the first and second derivatives, and use them to find all the usual information about the graph. In addition, find the horizontal and vertical asymptotes. Present your work in an organized and clear format.

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4. At 12:00 noon, a boat sailing east at 12 miles per hour crosses paths with a boat sailing north at 9 miles per hour. How fast is the distance between the two boats changing 2 hours later (assuming that their speeds and directions don't change)?

5. Find the point on the line $y = x$ that is closest to the point $(1, 3)$, and show that the line from that point to $(1, 3)$ is perpendicular to $y = x$. (Hint: Express the distance from the point (x, x) to the point $(1, 3)$ as a function of x.)

6. Here are a few more derivative problems, of the type that might appear on the test. Compute the following derivatives:

a) $\dfrac{d}{dx}\tan(x\ln(x))$ **b)** $\dfrac{d}{dt}\dfrac{t}{\sin^{-1}(t^2)}$ **c)** $\dfrac{d}{dz}2^z3^{1-z}$

d) $\dfrac{d^{42}}{dx^{42}}10^x$ **e)** $\dfrac{d}{ds}\ln\left(\tan^{-1}(s^2)\right)$ **f)** $\dfrac{d}{dx}x^{e^x}$

7. Use implicit differentiation to find $\dfrac{dy}{dx}$ as a function of x and y, if:

a) $xy^5 + 2x^5y = 3$ **b)** $\tan^{-1}(xy) = y$