This is the final lab for the course. Problems 1 through 8 and 10 are from Math 130 exams that I have given in the past.

You should turn in problems 10, 11, and 12 at the end of the lab for grading. *Work on other problems if you have time. An answer sheet will be available tomorrow in class.*

1. Compute the following derivatives

a) $\dfrac{d}{dx}(x^5 + x^3 + x) \cdot \ln(x)$

b) $\dfrac{d}{dx}\left(2x^3 + \sin(2x)\right)$

c) $\dfrac{d}{dx}\sin(xe^x)$

d) $\dfrac{d}{dx}x^2 \tan x$

e) $\dfrac{d}{dx}\dfrac{\sqrt{x}}{2x+1}$

f) $\dfrac{d}{ds}\sin(\pi s)\cos(\pi s)$

g) $\dfrac{d}{dx}\left(ax^2 + bx + c\right)^9$, where a, b, and c are constant.

h) $\dfrac{d}{dx}\cos(x^2)e^{\sin(x^2)}$

2. Let $f(x) = 3x^2 + 7$. Use the definition of the derivative, in terms of a limit, to show that $f'(x) = 6x$. The derivative, $f'(x)$, at a particular x-value represents the slope of a tangent line. Explain carefully why it is necessary to use a "limit" to calculate this slope.

3. Let $f(x)$ be defined by $f(x) = \begin{cases} x+1, & \text{for } x <= 0 \\ x, & \text{for } 0 < x < 1 \\ 2x-1, & \text{for } x \geq 1 \end{cases}$.

a) Compute the values of the following one-sided limits:

$$\lim_{x\to 0^-} f(x) = \_\_\_\_ \qquad \lim_{x\to 0^+} f(x) = \_\_\_\_$$

$$\lim_{x\to 1^-} f(x) = \_\_\_\_ \qquad \lim_{x\to 1^+} f(x) = \_\_\_\_$$

b) Does $\lim\limits_{x\to 0} f(x)$ exist? Why or why not?

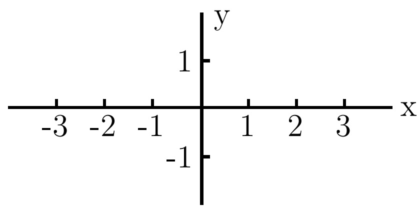
c) Does $\lim\limits_{x\to 1} f(x)$ exist? Why or why not?

d) Does $f'(1)$ exist? Why or why not?

4. Find all the critical points of the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$ and decide whether each critical point is a local maximum, a local minimum, or a point of inflection.

5. On the axes below, sketch the graph of a function $f(x)$ that satisfies all the following properties:

(1) f is continuous at 0 (2) $f'(0)$ does not exist
(3) f is decreasing for $x < 0$ (4) f is increasing for $x > 0$
(5) f has a point of inflection at the point $(x, y) = (1, 1)$.



6. A farmer wants to fence in a small rectangular enclosure and divide it in two with a fence down the middle. The fence around the outside of the enclosure will cost \$3.00 per foot, while the fence down the middle will only cost \$2.00 per foot. If the farmer will spend \$720.00 on fencing, what is the maximum area that she can enclose?

7. A car starts from rest and accelerates at a constant rate of 8 ft/sec^2. How long does it take the car to travel 100 feet?

8. Let $f(x) = x^3 + 3x^2 - 9x + 2$. Determine where $f(x)$ is increasing and where it is decreasing, and find any relative maxima or minima. Also determine where $f(x)$ is concave up and where it is concave down, and find any points of inflection. Show your work. Graph the curve $y = f(x)$ on the axes provided.

9. Verify that for any constant a, $\displaystyle\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \ln(x + \sqrt{a^2 + x^2}) + C$

10. A street lamp is 20 feet tall. At a certain instant, a rock is dropped from a point 20 feet above the ground and 10 feet from the lamp. (It falls with an acceleration due to gravity of -32 ft/sec.) How fast is the shadow of the rock moving along the ground one second after it is dropped?

11. When we work with trigonometric functions in Calculus, we use radians, not degrees. The reason is that derivative formulas are simpler when radians are used. Let $R(x)$ be the number of radians in x degrees. Then, for example, the sine of x degrees can be written as $\sin(R(x))$. Find a formula for $R(x)$, and use the chain rule to compute the derivative $\frac{d}{dx}\sin(R(x))$. This is the formula that you would have to use for the derivative of $\sin(x)$, if x were measured in degrees.

12. Based on graph shown at the right, find a numerical estimate of $f''(1)$. Use the fact that f'' is the derivative of f'. You won't be able to get a very accurate answer, but you should try to be as accurate as possible. **Carefully explain your reasoning.**