Please hand in your responses to the following exercises in class next Monday, September 13. You should work on these problems in a group of three or four students. The first three problems are from Chapter 1. The rest are about instantaneous velocities and limits.

1. Find values for A, B, C, and D so that the transformed sine function

$$y = A\sin(B(x - C)) + D$$

has the following properties: The minimum value of the function is 1 and the maximum value is 5. The period is 6π. When x is 0, y is 1. (There is more than one possibility for C.)

2. We know that $2^a \cdot 2^b = 2^{a+b}$ (which is obvious for positive integers a and b but is true for any numbers). We also know that $2^{\log_2(c)} = c$ (because of the definition of $\log_2(x)$ as the inverse of 2^x). Use these two facts to write a simple proof that $\log_2(AB) = \log_2(A) + \log_2(B)$. Hint: Write $A = 2^{\log_2(A)}$ and similarly for B.

3. $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. This means that the inverse of a sequence of operations consists of the inverse operations applied in reverse order. For example, let $f(x) = 3x + 1$. The operations applied to x are: multiply by 3, add 1. The inverse operations in reverse order are: subtract 1, divide by 3. Thus, the inverse of f is given by $f^{-1}(x) = (x - 1)/3$. Now, consider $g(x) = \log_4(3^{\sqrt[3]{x}/7} + 12)$. List the sequence of operations that is applied to x in this formula. Use your list to find a formula for $g^{-1}(x)$.

4. Limits are used to compute instantaneous velocities, but there are also other types of limits. For example, it's possible to ask what happens to the value of a function $f(x)$ "in the limit" as x gets bigger and bigger. Sometimes, the answer is a little surprising. Consider the function $f(x) = \sqrt{x^2 + x} - x$, For this function, make a table showing the value of the function for $x = 1$, $x = 10$, $x = 100$, $x = 1000$, and $x = 10000$. Based on the table, what happens to the value of $f(x)$ in the limit as x approaches infinity?

5. The position of an object at time t is given by $f(t) = 2t^2 - \frac{1}{t}$, for $t > 0$. Find a formula for the average velocity of the object on the interval between the times $t = 1$ and $t = 1 + h$, and simplify the formula. Use the simplified formula to find the instantaneous velocity at time $t = 1$.

6. The position of an object at time t is given by a function $f(t)$. The values of $f(t)$ are measured for certain values of t, as shown in the table below. The results are shown in the following table.

t	1.0	1.25	1.5	1.75	2.0
$f(t)$	1.5	4.0	5.6	6.0	5.0

Given the data in this table, you can't be sure what the instantaneous velocity at $t = 1.5$ is. But you can still give an estimate. Possible estimates could be $\frac{f(1.75)-f(1.5)}{1.75-1.5}$ or $\frac{f(1.5)-f(1.25)}{1.5-1.25}$, but these are not the **best** estimates that can be given based on the available data. Why not? How could you get a better estimate? What is your best estimate for the instantaneous velocity of the object at $t = 1.5$? Explain! (There is no perfect solution to this problem.)

7. Suppose that you manage to measure the position and velocity of an object at time $t = 3.7$. The position is 8.7, and the velocity is 1.2. Estimate the position of the object at time $t = 4$. Explain why your estimate is only an approximation. Is there any particular kind of motion for which your estimate would be exactly correct?

8. Suppose that its possible to "teleport" an object instantaneously from one point to another. (This might really be possible in quantum mechanics.) Let's say that the position of the object is 2 for time $t < 1$ and is 3 for time $t > 1$. Compute the value of the average velocity of the object for several small intervals that contain $t = 1$. What happens as the size of the intervals shrinks towards zero? Does the object have an instantaneous velocity at $t = 1$? What is the instantaneous velocity of the object for $t \neq 1$?

9. Recall that any interval, no matter how small, contains both rational numbers and irrational numbers. Suppose that the position of an object at time t is given the famous "Dirichlet function"

$$f(t) = \begin{cases} 0, & \text{if } t \text{ is a rational number} \\ 1, & \text{if } t \text{ is an irrational number} \end{cases}$$

(This is certainly not very likely—not even possible!) Explain why this function does not have an instantaneous velocity at **any** value of t? (Look at the possible average velocities on small intervals.)